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Endogenous Horizontal Mergers in Homogeneous Goods Industries with Bertrand Competition

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Abstract

We discuss the effect of horizontal mergers in homogeneous goods industries when firms compete à la Bertrand with increasing marginal costs of production. We set up a two-stage game where in the first stage firms decide whether to join the merger or to remain outside and in the second stage market competition takes place. We identify necessary and sufficient conditions for a market structure where a merger did occur to be coalition proof. We find that such market structure could be consumer surplus enhancing as it could arise even for lower post-merger prices with respect to the pre-merger scenario. This is in sharp contrast with the findings under both price and quantity competition where, absent efficiency gains, mergers unambiguously harm consumers.

Keywords: Homogeneous Goods; Horizontal Mergers; Bertrand Competition; Coalition Proof Nash Equilibrium.

JEL codes: C72; D43; G34; L13.

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1 Introduction

Mergers and acquisitions represent one of the main tools adopted worldwide by companies to increase their competitiveness. In the four years preceding the pandemic, on average, roughly 50.000 M&A deals have been completed globally. In between 2016 and 2019 the average value of the M&A transactions accounted for approximately 3500 U.S. billion dollars with a peak of 4800 U.S. billion dollars in 2015. These deals need a careful evaluation by the competent authorities, in order not to risk approving anticompetitive agreements which will ultimately harm consumers.

The literature has shown that the final effect of a merger depends on several factors like the type of competition, firms' technology, the demand shape, and so on. Another well-known result in oligopoly theory is the Bertrand paradox, which states that in a linear, homogeneous product price-setting game with identical and constant marginal cost of production across firms, the unique equilibrium entails all competitors setting price equal to marginal cost (*i.e.* a zero profit condition).

Surprisingly enough, starting from the seminal contribution by Deneckere and Davidson (1985), the literature on mergers in price setting oligopolies has mainly developed under the assumption of a linear technology. Thus, as soon as goods homogeneity is taken into account, the paradox kicks in and makes that all mergers but the one to monopoly are weakly profitable. In particular, unless all firms join the coalition and make positive profits, in all the remaining market structures these make zero profits before and after the merger.

However, the paradox is typically not observed in real life and the literature attempted to get rid of it in order to reconcile with empirical evidence. The seminal contribution by Kreps and Sheinkman (1983) shows that if firms first select a capacity level, and then price competition takes place under such production constraint, the outcome is the same as Cournot and the paradox is eliminated.

If on the one hand this feature surely reflects the behavior of several markets, on the other hand, such constraint can sometimes be drastic, as the Kreps and Sheinkman (1983) model implicity assumes that producing beyond capacity is infinitely costly. As stated by Besanko *et al.* (2010), it is often the case that *"In the real world hiring temporary workers, adding shifts, or expediting material deliveries to alleviate capacity constraints are common and often costly"*. In other words, capacity constraints are *soft* in the sense that production can in principle be adjusted quite rapidly but at a progressively higher cost.

The same principle is discussed in Cabon-Dhersin and Drouhin (2014; 2020) where firms first select a fixed production factor and in a second stage select another variable production factor in order to match demand and compete in prices. Thus, although the fixed factor can be seen as a capacity, firms can always produce beyond such level but at an increasing marginal cost.[1]

A case in point is the North American brick industry, where a considerable softening of capacity is achieved by installing an extra kiln, which however turns out to be very costly in case capacity exceeds demand due to its rapid deterioration (Wood, 2005). Additionally, such market

[1]Cost convexity is obtained by the decreasing marginal productivity of the second-stage variable factor.

experienced a strong consolidation. For instance, in 2000 General Shale acquired Cherokee Sandford Groups for 81 U.S. million dollars; in 2004 HANSON announced the acquisition of Athens brick CO. for roughly 40 U.S. million dollars; and in 2021 Sandford Groups acquired Meridian Brick.[2]

Thus, in this paper, we discuss the effect of horizontal mergers in a price setting oligopoly for a homogeneous good with increasing marginal cost of production. Furthermore, prior to competing à la Bertrand, we let firms explicitly decide whether or not to join a coalition. As typically such a decision is thoroughly discussed and a declaration of intent to merge cannot be a binding agreement for any subset of firms, we employ the Coalition Proof Nash Equilibrium (Bernheim *et al.*, 1987 - CPNE henceforth) as equilibrium concept of the merger formation stage.

Our main result is that a coalition proof market structure where a merger did occur in the first stage could be consumer surplus enhancing as a post-merger price strictly below the one in the pre-merger scenario could be charged in equilibrium. This is in sharp contrast with *i)* the general findings under quantity competition by Farrell and Shapiro (1990), who show that, absent efficiency gains, a merger unambiguously implies a post-merger price increase; and *ii)* with the findings obtained so far under price competition.

We also show that, if on the one hand merger profitability (*i.e.* a merged entity gaining no less than the sum of the merging parties pre-merger payoffs) has been a typical criterion to justify the occurrence of a merger, on the other hand, it is only one of the conditions that shape the incentives to endogenously join a coalition. In this regard, we show that the external stability condition à la D'Aspremont *et al.* (1983) also plays an important role.

As said, mergers and acquisitions are key strategic tools for firms. Two effects associated with M&A's have traditionally been identified. As reported in the U.S. merger guidelines such distinction is between coordinated and unilateral effects (see Asker and Nocke, 2021 for a detailed discussion). The first refers to the impact of a merger on the incentives for tacit collusion. Even though the effect of a concentration on the incentives for tacit collusion is a complex issue, several papers have shown that mergers generally tend to increase the likelihood of collusion (see, for example, Compte *et al.*, 2002; and Vasconcelos, 2005).

Unilateral (or non-coordinated) effects refer to how a merged entity can exploit an increase in market power due to a reduction in competition subsequent to the merger itself. Central to the unilateral effects analysis is the well-known trade-off between the aforementioned market power and efficiency gains, namely the possibility of the merging firms to reduce the per unit production cost, so that to render mergers more socially desirable.[3]

Also, it is worth emphasizing the importance of cost convexity in merger models that has been pointed out by Perry and Porter (1985) in their critique to the linear cost model by Salant *et al.*

[2]Cabon-Dhersin and Drouhin (2014) point out that the retailing sector in general matches these features, where the space on the shelves is the chosen capacity and the number of employees to fill the shelves is the variable factor. If demand is superior to capacity, then additional employees are needed to fill the shelves more frequently.

[3]These synergies, in the sense of Farrell and Shapiro (1990), imply that the merged firm's post-merger marginal cost, evaluated at the combined pre-merger output of the merger partners, has to be less than the pre-merger marginal cost of any merger partner.

(1983). In particular, Perry and Porter (1985) argue that the Salant *et al.* (1983) model should be viewed as a ”lock-up” model, rather than a proper merger model. This is because the new firm has no access to the combined productive capacity of the merger partners, since the unique effect of a k-firm merger is shutting down $k-1$ plants. In other words, there is no structural change between the merged entity and each merging firm, not in line with the observation that entities resulting from a merger are typically rather complex organizations (Huck *et al.*, 2004).

When instead costs are convex and a merger takes place, the new entity does not shut down all plants but one but rather shares its production among the plants in such a way that the marginal costs are equalized. This output rationalization effect makes that the new entity can experience a cost advantage with respect to the outside firms, which in turn could lead to a larger market share. Noteworthy, the advantage from this output rationalization effect is not the same as one of a merger-induced synergy. In our setup, in fact, if the pre-merger aggregate quantity of the merging firms equals (resp. exceeds) the one of the merged entity, such a quantity is produced at an equal (resp. larger) cost. This crucial aspect of mergers has been highlighted by Ivaldi and Verboven (2005) when quantifying their welfare effects.

As pointed out, the literature on horizontal mergers under Bertrand competition and homogeneous products is relatively sparse. The main reference is surely Deneckere and Davidson (1985). In their model where firms produce differentiated (and homogeneous as a limit case) products with a linear and identical cost function and simultaneously compete in price, they show that *i)* mergers are always (weakly or strictly) profitable; *ii)* larger coalitions generate (weakly or strictly) more profits, and *iii)* both the insiders' and the outsiders' prices are larger in the post-merger scenario, so that consumers are unambiguously worse off.[4]

As mentioned above, this strand has remained unexplored for many years, and only very recent contributions can be found. One of the few exceptions is Chen and Li (2018) who show that in a capacity constrained, homogeneous goods, price competition model (like the Edgeworth-Bertrand model), when a binding capacity constraint turns into a slack one as a consequence of a merger, then it implies a market price increase.

Wang and Zhao (2021) extend Deneckere and Davidson (1985) by allowing for asymmetric linear costs of production and for the possibility for the merged entity to transfer the most efficient technology to all the other plants at no additional cost. They show that the overall effect of the merger on outside firms' profits and consumer surplus is ambiguous and depends on the magnitude of the merger-induced synergies, whereas insiders are always better off. [5]

Our model thus differs from the aforementioned contributions fundamentally on several grounds:

[4]This created a clear separation with the quantity competition and homogeneous products model of Salant *et al.* (1983), where mergers are profitable only if these involve at least the 80% of the market.

[5]The economic literature has also started to debate the effects of horizontal mergers on innovation incentives (see the survey in Kokkoris and Valletti, 2020). Another interesting example is Motta and Tarantino (2021) where firms simultaneously invest in a cost reducing R&D investment and in prices. Although differently from the constant marginal cost case of Deneckere and Davidson (1985) the final effect, ignoring efficiency gains, on the outsiders' prices is *a priori* ambiguous, they show that mergers are still anticompetitive (*i.e.* reduce consumer surplus) as long as a demand system satisfies the Independence of Irrelevant Alternatives (IIA). After that, they show parametric results for commonly adopted demand systems which do not satisfy the IIA, and still mergers reduce consumer surplus.

first, we study a Bertrand model with increasing marginal costs of production; second, we focus specifically on homogeneous goods industries. In addition, instead of imposing the presence of a coalition, we endogenize its size by letting firms decide whether or not to merge in a simultaneous and non-cooperative game, prior to competing à la Bertrand. Furthermore, it seems natural to assume that the decision to be part of a coalition would be thoroughly discussed among firms and that a declaration of intent to merge cannot be a binding agreement, namely a unique or multiple firms may ultimately decide to abandon such option. In this regard, we employ the Coalition Proof Nash Equilibrium (Bernheim *et al.*, 1987 - CPNE henceforth) as equilibrium concept of the merger formation stage, where a strategy profile must be robust to unilateral and multilateral *self-enforcing* deviations.[6]

Thoron (1998) adopted the same approach in a one-stage game where the stability of cartels is analyzed. However, the restrictions imposed on the market structure, although suitable in a collusive environment, may well not hold in a merger game. In particular, Thoron (1998) assumes that the profit of an outsider always increases with the size of the coalition and that the existence of a coalition generates a positive externality on the outsiders in such a way that they are always free riders.

A more recent contribution related to ours is Cabolis *et al.* (2021) who set up a two-stage, three-firm model which simultaneously decide the amount of cost reducing R&D investment and whether or not to join a merger prior to competing à la Cournot in a homogeneous product market. The important touchpoint is that the same equilibrium concept is used to determine the size of the coalition that will endogenously arise.[7] However, to the best of our knowledge and as Cabolis *et al.* (2021) themselves acknowledge, such a concept that seems to naturally capture relevant aspects of a merger formation process has been adopted in a merger game for the first time in their work.

The rest of the paper is structured as follows: in Section 2 we present the baseline model; in Section 3 we discuss merger profitability; in Section 4 we endogenize the merger formation process; in Section 5 we provide the market equilibrium analysis by showing the conditions for a coalition proof market structure, and a welfare analysis. Section 6 concludes. All proofs are in the Appendix.

2 The Model

Consider a market in which N symmetric firms produce a homogeneous good and let $\mathcal{N} = \{1, 2, ..., i, ..., N\}$ be the index set of the firms. Firms face the direct demand $q = f(p)$, where $q \equiv \sum_{i=1}^{n} q_i$ is the total output produced in the industry at price p and q_i is firm i's production,

[6]The Nash equilibrium is instead adopted in the Bertrand stage, where communication on joint action (*i.e.* price coordination) is prohibited.

[7]Several papers though have considered endogenous coalition formation games. For instance, Pesendorfer (2005) develops a dynamic infinite-horizon game where the equilibrium concept used is the Markov perfect Nash equilibrium. Vasconcelos (2006) uses a coalitional stability concept with fully farsighted players in a Cournot setting. An alternative approach is treating merger formation as a cooperative game like in Horn and Persson (2001).

with $i = 1, 2, ..., N$. The demand is downward sloping, twice continuously differentiable, strictly concave and there exists a price p^{max} such that $q(p^{max}) = 0$, and a q^{max} such that $p(q^{max}) = 0$. Each firm supplies q_i units of output as follows:

$$q_i(\boldsymbol{P}) = \begin{cases} 0 & \text{if } p_i > p_j \text{ for some } j \in \mathcal{N} \\ \frac{q(p_i)}{N_=} & \text{if } i \in N_= \\ q(p_i) & \text{if } p_i < p_j \ \forall j \neq i, \ j \in \mathcal{N} \end{cases} \tag{1}$$

where $\boldsymbol{P} \equiv (p_1, p_2, ..., p_i, ..., p_N)$ and $\mathcal{N}_= \subseteq \mathcal{N}$ is the index subset of firms setting the same and lowest price, with cardinality $N_=$.

From the above formulation, it follows that when a firm undercuts the rivals it covers the entire market. This assumption can be justified by the presence of hidden costs of avoiding customers away (Dixon, 1990) or due to a regulatory argument in which firms are forced to match the entire demand in order not to harm consumers (Spulber, 1989). Each firm produces q_i units according to the quadratic cost function $C_i(q_i) = \frac{q_i^2}{2}$.

Also, let:

$$\hat{\pi}_i(p, N_=) = \frac{pq(p)}{N_=} - \frac{1}{2}\left(\frac{q(p)}{N_=}\right)^2$$

and

$$\pi_i(p) = pq(p) - \frac{q(p)^2}{2}.$$

In words, $\hat{\pi}_i(p, N_=)$ represents firm i's payoff when equally splitting the market among those firms setting the lowest price in the industry, whereas π_i is firm i's payoff as the unique firm setting the lowest price in the industry. Thus, firm i's payoff is:

$$\Pi_i(\boldsymbol{P}) = \begin{cases} 0 & \text{if } p_i > p_j \text{ for some } j \in \mathcal{N} \\ \hat{\pi}_i(p_i, N_=) & \text{if } i \in N_= \\ \pi_i(p) & \text{if } p_i = p < p_j \ \forall j \neq i, \ j \in \mathcal{N} \end{cases} \tag{2}$$

where we assume that both $\hat{\pi}_i(p_i, N_=)$ and $\pi_i(p)$ are strictly concave functions in p_i.
A Bertrand-Nash equilibrium is a vector of prices $\boldsymbol{P}^* = (p_1^*, p_2^*, ..., p_i^*, ..., p_N^*)$ such that:

$$\Pi_i\left(p_1^*, p_2^*, ..., p_i^*, ..., p_N^*\right) \geq \Pi_i\left(p_1^*, p_2^*, ..., p_i', ..., p_N^*\right), \ \forall p_i' \neq p_i^* \ and \ \forall i \in \mathcal{N}.$$

It turns out that the above Bertrand game may possess a continuum of equilibria in pure strategies (Dastidar, 1995), in which each firm is assigned an equal share of the market.[8] In particular, given the symmetry across firms, the set of equilibrium prices $\mathcal{P}^B$ is described in Proposition 1

[8]The main intuition is that, differently from the linear case, where the gain from slightly undercutting a rival and serving the market is always larger than the cost increase of doing so (thus leading to the paradoxical equilibrium

in Dastidar (1995) and it is given by $\mathcal{P}^B = \left[\underline{p}(N), \overline{p}(N)\right]$, where the lower bound $\underline{p}(N)$ is the unique (see Lemma 1 in Dastidar, 1995) solution with respect to p of:

$$\frac{pq(p)}{N} - \frac{q(p)^2}{2N^2} = 0 \tag{3}$$

and the upper bound $\overline{p}(N)$ is the unique (see Lemma 5 in Dastidar, 1995) solution to:

$$\frac{pq(p)}{N} - \frac{q(p)^2}{2N^2} = pq(p) - \frac{q(p)^2}{2}. \tag{4}$$

Equation (3) captures the fact that in any equilibrium, firms equally share the market, as undercutting must be unprofitable, and that the lowest equilibrium payoff corresponds to the one where firms obtain zero profits by setting price equal to average variable costs.[9] The same reasoning applies to the upper bound, where the maximum equilibrium payoff that a firm would obtain by equally splitting the market with its competitors equals a monopolistic (and positive) payoff.[10]

Alternatively, given our general demand framework, it will prove useful to express the set of equilibria in terms of the price-quantity ratio $\frac{p}{q(p)}$ and refer to the equilibrium price-quantity ratio set $\mathcal{R}(p)$.[11] This allows us to reframe our analysis in terms of the symmetric number of firms and the number of merging firms only. It is straightforward to show that such a set is given by $\mathcal{R}^B(N) = \left[\underline{\mathcal{R}}^B, \overline{\mathcal{R}}^B\right] = \left[\frac{1}{2N}, \frac{N+1}{2N}\right]$ in the pre-merger scenario, where B stands for *before*.

We note that Dastidar (1995) does not consider whether the equilibrium price is larger than the joint profit maximization price. We believe though that this possibility should be ruled out because firms could obtain the same profit by charging a lower price that would surely be less likely to attract the antitrust authorities' attention.

Assumption 1. $\overline{\mathcal{R}}^B < \frac{\bar{\bar{p}}}{q(\bar{\bar{p}})}$ *where* $\bar{\bar{p}} := \arg\max_p k\hat{\pi}_i(p, N)$.

3 Merger Analysis

In this section we shall explore the possibility that $2 \leq k < N$ firms merge into a new entity *m*.[12] One effect induced by the merger is the transition from an N-firm to an $(N-k+1)$-firm market structure, which is formally described by the partition (or *coalition structure*) of $\mathcal{N}$,

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with firms getting zero profits), in a convex cost setting the strength of these effects is reversed for a whole set of prices.

[9] A slight increase would still yield zero profits, a slight undercut would imply negative benefits.

[10] Without considering the effect of a merger, an interesting result is that, under convex variable costs the per-firm Bertrand payoff may be larger than the Cournot payoff. Moreover, whenever this happens, social welfare is lower under Bertrand competition (Delbono and Lambertini, 2016).

[11] Since $q(p)$ is monotonically decreasing, then $\frac{p}{q(p)} = t,\ t > 0$ has a unique solution in p. Thus, the existence of a result in terms of the price-quantity ratio implies being able to re-propose the same result in terms of p only. Furthermore, $\frac{p}{q(p)}$ is a monotone and increasing function in p.

[12] We exclude mergers to monopoly. Firms rarely propose mergers to monopoly as the government is virtually certain to challenge such mergers.

$\triangle = \{\mathcal{M}, k{+}1, ..., N\}$, where $\mathcal{M} \subset \mathcal{N}$ is the subset of indexes of the merging firms.[13] Moreover, differently from the linear cost case in which the new entity can optimally shut down $k-1$ plants, under convex costs, the production of the merged entity is optimally shared across the k plants in such a way that all marginal costs are equalized.[14] It follows that the output produced in each plant is $\frac{q_m}{k}$, and the total cost borne by the entity is $C_m(q_m) = \frac{(q_m)^2}{2k}$. As pointed out, this output rationalization across plants is not the same as an efficiency gain, namely a general improvement in the per-unit cost of production. If the cumulated output of the merging parties kq_i was identical to the one of the merged entity q_m, the overall cost would be the same.

However, since in the post-merger scenario the cost structures differ across firms, the pre-merger symmetry is broken. In this case, the set of equilibrium prices is described by Lemma 9 in Dastidar (1995) and it is $\mathcal{P}^A = \left[\max_{i\in\mathcal{N}^A} \underline{p}_i, \min_{i\in\mathcal{N}^A} \overline{p}_i\right]$, where A stands for *after* and $\mathcal{N}^A$ denotes the post-merger index set of firms. Its lower bound is the unique solution with respect to p to:

$$\frac{pq(p)}{N-k+1} - \frac{q(p)^2}{2(N-k+1)^2} = 0, \tag{5}$$

whereas the upper bound is the unique solution to:

$$\frac{pq(p)}{N-k+1} - \frac{q(p)^2}{2k(N-k+1)^2} = pq(p) - \frac{q(p)^2}{2k}. \tag{6}$$

Solving (5) and (6) for $\frac{p}{q(p)}$, one obtains that the post-merger equilibrium price-quantity ratio set $\mathcal{R}^A(N,k) = \left[\underline{\mathcal{R}}^A, \overline{\mathcal{R}}^A\right] = \left[\frac{1}{2(N-k+1)}, \frac{N-k+2}{2k(N-k+1)}\right]$. At this point we make the following assumption:

Assumption 2. $k \leq \frac{n+2}{2}$.

Assumption 2 ensures that $\overline{\mathcal{R}}^B > \underline{\mathcal{R}}^B$, which in turn makes sure that the merged entity is not able, in equilibrium, to cause the market foreclosure of the outside firms, by charging a sufficiently low price.

3.1 Merger Profitability

A typical criterion that has been often adopted in the literature to justify the occurrence of a merger is its profitability. In particular, a merger is said to be profitable if the payoff of the merged entity π_m is larger than the cumulated pre-merger payoffs of its members. Formally, we

[13] Clearly the pre-merger scenario is described by the finest partition of $\{1, 2, ..., i, ..., N\}$. Also, given the pre-merger symmetry across firms, without loss of generality, we refer to a coalition structure with a k -firm merger and $N-k$ outsiders as the one where the first k indexes in $\mathcal{N}$ are those forming the coalition.

[14] In the linear cost case every output redistribution among the k plants would yield the same cost for the entity. Thus, it is typically assumed that after the merger all the production is concentrated in a single plant.

need that:

$$\underbrace{\frac{p^A q(p^A)}{N-k+1} - \frac{q(p^A)^2}{2k(N-k+1)^2}}_{\frac{p^A}{q(p^A)} \in \mathcal{R}^A(N,k)} > \underbrace{k\left(\frac{p^B q(p^B)}{N} - \frac{q(p^B)^2}{2N^2}\right)}_{\frac{p^B}{q(p^B)} \in \mathcal{R}^B(N)}. \tag{7}$$

As it will be pointed out, a mere profitability analysis is not in general sufficient to justify the occurrence of a merger once the decision to whether or not join a coalition is strategically made by firms. However, in our analysis, it will be a necessary condition, so that we believe it is worth including our findings in the main body of the paper.

At this point we are ready to summarize our findings in the next proposition.[15,16]

Proposition 1. *Let* $I(n,k) = \frac{1}{2}\left(\frac{1}{N} + \frac{1}{k(N+1)-k^2}\right)$ *be the unique intersection point between* π_m *and* $k\hat{\pi}_i(p,N)$ *and let* $h(N) = \frac{1}{2}\left(1+\sqrt{1+4N}\right)$*. Then a k-firm merger is strictly profitable:*

i) $\forall \frac{p^A}{q(p^A)} \in \mathcal{R}^A$*, if* $k \geq h(N)$ *and* $\frac{p^B}{q(p^B)} \in \left[\underline{\mathcal{R}}^B, \frac{\tilde{p}}{q(\tilde{p})}\right)$*, for some* $\frac{\tilde{p}}{q(\tilde{p})} \in \mathcal{R}^B$*;*

ii) $\forall \frac{p^A}{q(p^A)} \in \left[I(N,k), \overline{\mathcal{R}}^A\right]$*, if* $2 \leq k < h(N)$ *and* $\frac{p^B}{q(p^B)} \in \left[\underline{\mathcal{R}}^B, \frac{\tilde{p}}{q(\tilde{p})}\right)$*, for some* $\frac{\tilde{p}}{q(\tilde{p})} \in \mathcal{R}^B$*;*

iii) $\forall \frac{p^A}{q(p^A)} \in \left[\underline{\mathcal{R}}^A, I(N,k)\right)$*, if* $2 \leq k < h(N)$ *and* $\frac{p^B}{q(p^B)} \in \left[\frac{p^A}{q(p^A)}, \frac{\tilde{p}}{q(\tilde{p})}\right)$*, for some* $\frac{\tilde{p}}{q(\tilde{p})} \in \mathcal{R}^B$ *and* $\forall \frac{p^B}{q(p^B)} \in \left[\underline{\mathcal{R}}^B, \frac{p^A}{q(p^A)}\right)$.[17]

As shown in Lemma 1 in the Appendix, point *i)* tackles cases where $\mathcal{R}^A$, which is always nested into $\mathcal{R}^B$, completely lies to the right of the intersection point $I(N,k)$, whereas points *ii)-iii)* tackle cases where $I(N,k) \in \mathcal{R}^A$.[18] The common part of points *i)* and *ii)* is that whenever a price compatible with a post-merger price-quantity ratio strictly larger than $I(N,k)$ is charged, then the pre-merger price-quantity ratio must necessarily be below such value.

However, interestingly enough, whenever a price compatible with a post-merger price-quantity ratio strictly below the critical value $I(N,k)$ is charged, then a profitable merger could be obtained even for larger pre-merger price-quantity ratios. Hence, profitable mergers can imply an increase in consumer surplus, even in absence of efficiency gains. This result, as shown by Farrell and Shapiro (1990), cannot be obtained in quantity-setting oligopolies for general demand and cost functions and has not yet been obtained under price-competition as well.

Figures 1 and 2 provide a graphical intuition.

[15] The merger is weakly profitable if (7) holds with equality. Throughout the paper we will focus on strictly profitable mergers only.

[16] We also want to emphasize that in other contexts, firms could decide to strategically engage in non profitable mergers, in order for example, to preempt other mergers from the rivals, which would induce an even worse scenario for the non-merging firms (Fridolfsson and Stennek, 2010).

[17] The $\frac{\tilde{p}}{q(\tilde{p})}$'s in points *i)-iii)* are not in general the same. This abuse of notation is intended not to burden the presentation.

[18] We prove in Lemma 2 in the Appendix that $I(N,k)$ is the unique intersection point between π_m and $k\hat{\pi}_i$.

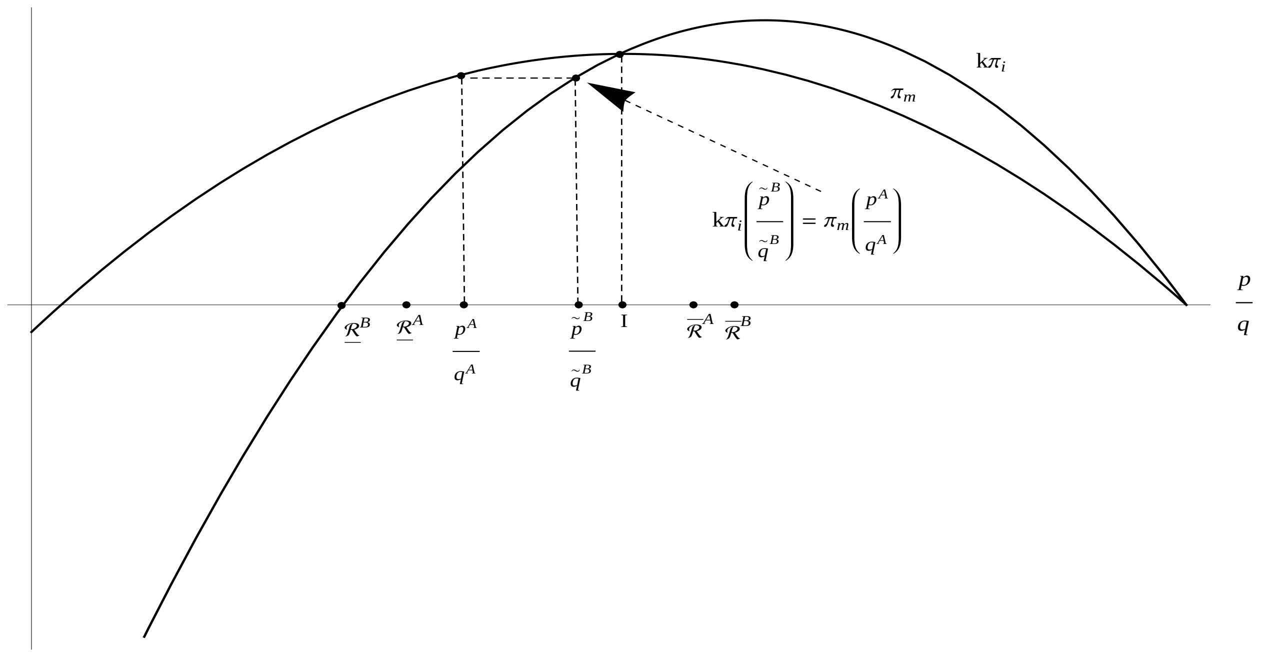

Figure 1: *The* $\underline{\mathcal{R}}^A < I(N,k)$ *case. For a given* $\frac{p^A}{q^A} \in \left[\underline{\mathcal{R}}^A, I(n,k)\right)$*, the locus of pre-merger price-quantity ratios* $\left[\frac{p^A}{q^A}, \frac{\tilde{p}^B}{\tilde{q}^B}\right)$ *ensure profitable mergers.*

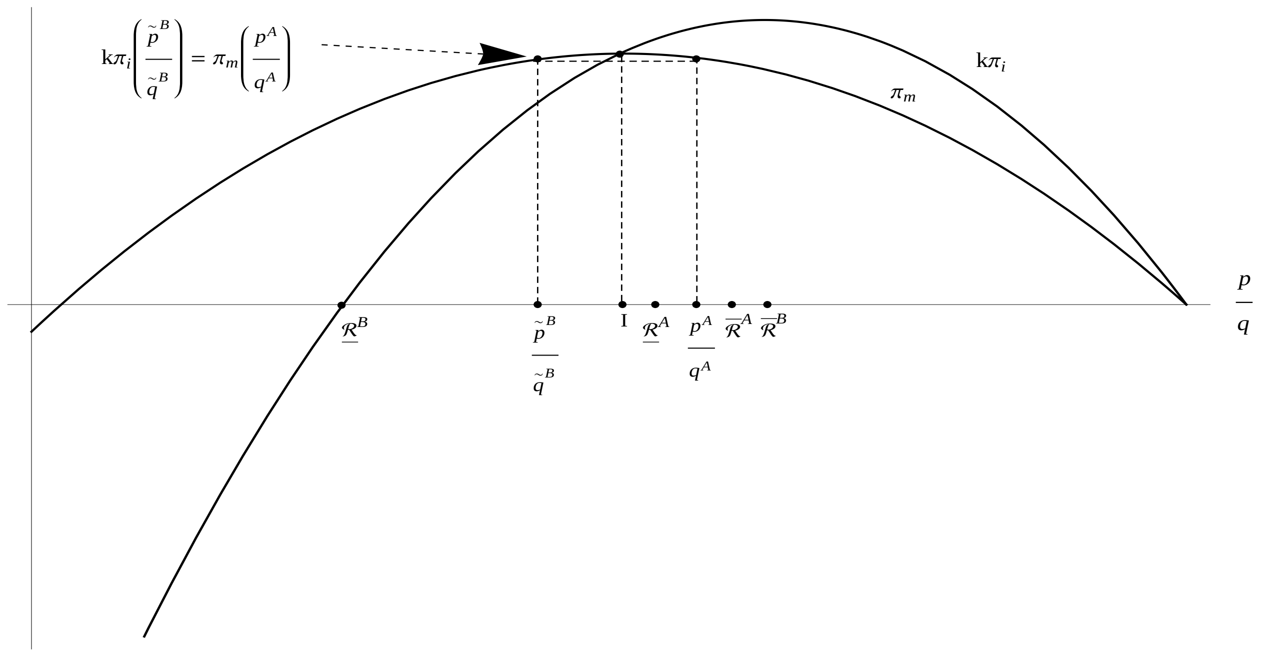

Figure 2: *The* $\underline{\mathcal{R}}^A > I(N,k)$ *case. For a given* $\frac{p^A}{q^A}$*, the locus of pre-merger price-quantity ratios ensuring profitable mergers must lie to the left of such point.*

Two effects come into play: first, although the merged entity produces more than a pre-merger firm, its production is strictly below the aggregate production of the insiders. Hence, equally sharing q_m across the k plants keeps the overall cost for the entity sufficiently low. Second,

$I(N,k) \in \mathcal{R}^A$ only when the number of merging firms is not too large, so that the per-insider payoff can be above the one in absence of a merger.[19]

4 Coalition Proof Market Structures

So far, we have discussed the conditions ensuring merger profitability. As a convenient and intuitive criterion in order to describe a successful merger, such a requirement has often been adopted as a proper justification for its occurrence. At the same time, however, it may neglect some important issues. Admittedly, it has little to do with an exhaustive concept of equilibrium once the decision to join a coalition is not exogenously imposed to a subset of firms.

In this regard, this section is dedicated to the analysis of an endogenous merger formation process. In particular, we will focus on a two-stage game where in the first stage each firm $i \in \mathcal{N}$ decides simultaneously whether to join the coalition or to remain outside.[20] Thus, each firm's action space is binary $\mathcal{A}_i = \{merge\ (M), not\ merge\ (NM)\}$. In the second stage, given the coalition structure from stage 1, the firms $i \in \mathcal{N}^A$ simultaneously select a price $p \in [0, +\infty)$.

Since merging is a major decision for firms and a challenging task affecting different aspects of management, it is natural to assume that *i)* firms have the possibility to discuss this merging option and try to reach agreements for joint action, and *ii)* an agreement (*i.e.* a decision to merge) is not binding in the merger-formation stage.[21] Therefore, we believe that an appropriate solution concept should take into account the possibility of deviations by coalitions of firms, as well as by individual firms in the merger formation stage.[22]

In this regard, a useful concept is the *Strong Nash Equilibrium* (SNE) (Aumann, 1959). A SNE is a strategy profile which is robust to deviations by any admissible coalition of players. The drawback of this concept is that it could be too strong, and it is often the case that it is impossible for a game to possess one. In order to overcome this issue, the somewhat milder concept of *Coalition Proof Nash Equilibrium* (CPNE) has been proposed by Bernheim *et al.* (1987), in which an equilibrium is only robust to *self-enforcing* unilateral and multilateral deviations.[23] In other words, once a deviation has occurred, only further deviations by any sub-coalition of the originally deviating players must be taken into account. Although such concept seems suitable in merger games, it was only recently introduced in this strand by Cabolis *et al.* (2021) in a model where firms invest in a cost reducing R&D investment and make merger proposals before competing à la Cournot. We adhere to their formal definition.

[19]We will show that this conclusion holds even when restricting attention to coalition proof market structures where a merger did occur in the first stage.

[20]We still maintain the assumption of a single coalition.

[21]Once a merger has occurred, it is appropriate to assume that such an agreement is binding. This formally implies that in the second stage of the game the deviations of the individual insiders must be overlooked and the merged entity is only allowed to deviate as a unique player.

[22]The Nash equilibrium is instead adopted in the second stage of the game as coordination on joint action (or price coordination) is generally prohibited.

[23]This, however, does not ensure the existence of a CPNE, which may well fail to exist (see Moreno and Wooders, 1996).

Definition 1. *Let* $G = [\{\pi_i\}_{i\in\mathcal{N}}, \{S_i\}_{i\in\mathcal{N}}]$, *with* $|\mathcal{N}| = N$ *be a simultaneous game with* N *players, individual payoff* π_i, *individual set of strategies* S_i *and* $\cup_{i=1}^{n} S_i = S$. *Let* C *be a coalition (or an index-subset) of the* N *players with cardinality* $|C|$, *let* $\mathcal{C}$ *be the set of coalitions and* $\cup_{i\in C} S_i = S_C$ *be the set of strategies of the members of coalition* C. *A strategy profile* $\boldsymbol{s}^* = (s_1^*, s_2^*, ..., s_N^*)$ *is a CPNE if no* $C \in \mathcal{C}$ *has a self-enforcing deviation, where the set of self-enforcing deviations SED*(S, c) *is recursively defined as follows:*

i) *if* $|C| = 1$, *then SED*$(S, c) = S_C$;

ii) *if* $|C| > 1$, *then SED*$(S, c) = \{\hat{s}_C \in S_C | \nexists\, \tilde{C} \subset C, \tilde{C} \neq \emptyset,\ s_{\tilde{C}} \in$ *SED*$(\tilde{S}_C, S_{N\setminus C})$ *such that* $\pi_i(s_{\tilde{C}}, \hat{s}_{\tilde{C}\setminus C}, S_{N\setminus C}) > \pi_i(\hat{s}_C, S_{N\setminus C})\}$;

In general, identifying a CPNE could be a very complicated task due to the large number of possible deviations. For example, in a simplified version of their three-firm model, Cabolis *et al.* (2021) need to consider fifteen possible deviations. Thus, in order to show whether our model admits a CPNE and highlight some of its features we reduce the number of firms to four and, as typical in the merger literature, we discuss the effect of a bilateral merger.

5 Equilibrium analysis

We now focus on the resolution of our two-stage game by backward induction. Since the sets of equilibrium prices for $(N = 4, k = \{0, 1, 2, 3\})$ have already been presented in Section 2, we are just left with discussing the merger formation process. Before proceeding we need the following definition as in D'Aspremont *et al.* (1983), and we let π_k^s denote the payoff of a firm when a k-firm merger took place, with $s = \{M, NM\}$.

Definition 2. *A merger is externally stable if* $\pi_k^{NM} > \pi_{k+1}^{M}$.

In words, a merger is externally stable if an outsider cannot improve his payoff by unilaterally entering a k-firm merger.

We now move to the analysis of the coalition proofness of the market structure $\{\mathcal{M}, 3, 4\}$. At this stage we let firms to simultaneously and non cooperatively announce whether they want to be part of the merger or remain outside.

In general, in order for a market structure to be a CPNE of the game, it must be invulnerable to all self-enforcing deviations by coalitions of one or more firms. In this case, excluding the analysis of further deviations by subcoalitions whenever necessary, we have five possible deviations.

First, either an insider unilaterally leaves the merger inducing the strategy profile $\{M, NM, NM, NM\}$ or an outsider unilaterally joins it inducing the strategy profile $\{M, M, M, NM\}$. These deviations are not beneficial (and thus also self-enforcing) if the merger is both profitable and externally stable in the sense of d'Aspremont *et al.* (1983), respectively.[24]

[24]It is worth noting that in the bilateral merger case, merger profitability is equivalent to the internal stability condition in the sense of d'Aspremont *et al.* (1983). Formally it requires that $\pi_k^M > \pi_{k-1}^{NM}$, namely an insider cannot obtain a larger payoff by unilaterally leaving the merger.

Concerning bilateral deviations, two possibilities are available: both insiders exit the merger inducing the strategy profile $\{NM, NM, NM, NM\}$, or an insider exits the merger and an outsider joins it inducing the strategy profile $\{M, M, NM, NM\}$.[25] Finally, concerning trilateral deviations, either two insiders exit the merger and an outsider joins it inducing the strategy profile $\{M, NM, NM, NM\}$, or one insider leaves the merger and two outsiders join it inducing the strategy profile $\{M, M, M, NM\}$.

Before presenting our results concerning the equilibrium market structures, let us adopt the following notation: p_k^A and p_i^B, with $i = \{0, 1\}$, denote respectively the price after a k-firm merger and the price before a coalition is formed (or alternatively when only a degenerate coalition of size 1 is formed).

Our results regarding the coalition proofness of a bilateral merger are summarized in the next proposition.

Proposition 2. *The market structure $\{\mathcal{M}, 3, 4\}$ is coalition proof if and only if the merger is profitable and externally stable.*

As already pointed out, it is often the case that profitability alone does not represent an exhaustive criterion to justify the occurrence of a merger.[26] Even in the simple case of a bilateral merger in a four-firm market, coalition proofness requires the merger being externally stable as well. These conditions are obtained from the two aforementioned unilateral deviations, as both bilateral and trilateral deviations will never take place.[27] Another observation is that the external stability condition imposes a refinement on the post-merger price-quantity ratios which can be sustained in a coalition proof market structure. This, in turn, could imply the elimination of those values compatible with a lower price-quantity ratio (*i.e.* with those values ensuring an increase in the consumer surplus) as shown in Section 2.[28] In this regard, we now move to our welfare analysis and show that, indeed, consumer surplus enhancing coalition proof market structures are possible.

To clearly show our point we focus on the linear demand specification. In particular we assume that $p(Q) = \max\{0, 1 - Q\}$. We obtain the following result:

Proposition 3. *In our linear demand specification, the market structure $\{\mathcal{M}, 3, 4\}$ is coalition proof and it is compatible with a lower post-merger price if and only if $p_2^A \in \left(\frac{1}{7}, \frac{5}{29}\right)$ and $p_i^B \in \left(p_2^A, \frac{5}{9} - \frac{2}{27}\sqrt{45 - 126 p_2^A + 117 (p_2^A)^2}\right]$, $i = \{0, 1\}$.*

Deneckere and Davidson (1985), assuming Bertrand competition with differentiated products find that a coalition raises price, which further increases as a reaction of the outsiders. In

[25] It is important to notice that, although both a unilateral and a bilateral deviation induce the same market structure with all independent firms $\{1, 2, 3, 4\}$, the two are different in terms of strategy profiles, as in one a firm still selects to M, whereas in the second one all firms select NM. This distinction is crucial when considering further deviations, if necessary, from a given market structure.

[26] The reader can find in the Appendix the mathematical expressions ensuring the coalition proofness of $\{\mathcal{M}, 3, 4\}$.

[27] Clearly restrictions on several price-quantity ratios are needed as a change in the first-stage strategy profile is associated with a different second-stage action space.

[28] Lower post-merger price-quantity ratios are possible since $k = 2 < 2.56 = h(4)$.

quantity-setting games, Farrell and Shapiro (1990) show that the same conclusion holds for general demand and cost functions. Thus, in a setting à la Perry and Porter (1985), social welfare (*i.e.* the sum of consumer surplus and industry profits) can increase only through strong enough cost synergies, or, as in McAfee and Williams (1992), when the market share of the outsiders is larger than the pre-merger shares of the merging firms.

However, it is often the case that authorities almost uniquely consider the variation in consumer surplus as relevant. In this regard, we show that a merger can be consumer surplus enhancing since it can endogenously occur for a lower post-merger price. Moreover, our analysis demonstrates that mergers can be pro-competitive even in cases where the merged entity has a significant market share. In terms of policy implications, the above analysis suggests that if the antitrust authorities are in the position to control a range of market prices, then a horizontal merger that takes place in a setting similar to the one considered here should be thoroughly analyzed.

6 Conclusions

We discussed the effect of horizontal mergers among price-setting firms producing a homogeneous product and facing increasing marginal costs of production. We modeled firms' interaction as a two-stage game where in the first stage firms simultaneously and independently decide whether to join a coalition or to remain outside, and in the second stage they compete à la Bertrand. We have shown when a market structure where a bilateral merger did occur is coalition proof. We have also shown that merger profitability is only one of the requirements for a coalition market structure to emerge, and that other concepts like the external stability à la D'Aspremont *et al.* (1983), also play important roles. Interestingly, we obtain that a coalition proof market structure where a merger did occur can emerge even though firms set a strictly lower price with respect to the pre-merger scenario. Consequently, consumers may well be better off as a consequence of a merger, which is in sharp contrast with the general findings by Farrell and Shapiro (1990) under quantity competition and those that so far have been obtained under price competition where, absent efficiency gains, a merger can take place only at the cost of lowering consumer surplus.

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Appendix

Proof of Proposition 1.

Let us start with the following lemmas.

Lemma 1. $\mathcal{R}^A(N) \subset \mathcal{R}^B(N,k)$.

Proof. The result follows since $\frac{1}{2N} < \frac{1}{2(N-k+1)}$ and $\frac{N-k+2}{2k(N-k+1)} < \frac{N+1}{2N}, \forall (N,k)$ with $k \leq \frac{2+N}{2}$. □

Lemma 2. *There exists a unique price-quantity ratio $I(N,k)$ such that $\pi_m = k\hat{\pi}_i(p,N)$. Moreover, if $\frac{p}{q(p)} > (<) I(N,k) \Rightarrow \pi_m < (>) k\hat{\pi}_i(p,N)$.*

Proof. The equality:

$$\pi_m = \frac{pq(p)}{N-k+1} - \frac{q(p)^2}{2k(N-k+1)^2} = k\left(\frac{pq(p)}{N} - \frac{q(p)^2}{2N^2}\right) = k\hat{\pi}_i(p,N) \tag{A-1}$$

can be rewritten as:

$$pq(p)\left(\frac{1}{N-k+1} - \frac{k}{N}\right) = q(p)^2\left(\frac{1}{2k(N-k+1)^2} - \frac{k}{2N^2}\right). \tag{A-2}$$

Dividing both sides by $q(p)^2$ and rearranging yields:

$$\begin{aligned}\frac{p}{q(p)} &= \left(\frac{N^2 - k^2(N-k+1)}{2N^2k(N-k+1)}\right)\left(\frac{N(N-k+1)}{N-k(N-k+1)}\right)\\ &= \frac{N+k(N-k+1)}{2Nk(N-k+1)} \equiv I(N,k).\end{aligned} \tag{A-3}$$

The second part of the statement follows by noticing that both $\frac{1}{N-k+1} - \frac{k}{N}$ and $\frac{1}{2k(N-k+1)^2} - \frac{k}{2N^2}$ are negative. □

Lemma 3. *If* $(N,k) = (3,2)$ *and* $k < \left(1+\sqrt{1+4N}\right)$*, then* $I(N,k) > \underline{\mathcal{R}}^A$*. If* $k > \left(1+\sqrt{1+4N}\right)$*, then* $I(N,k) < \underline{\mathcal{R}}^A$*.*

Proof. Omitted □

We now show whenever the merger is strictly profitable depending on $I(N,k) < (>)\underline{\mathcal{R}}^A$.

case 1: $I(N,k) < \underline{\mathcal{R}}^A$.

Pick any $\frac{p^A}{q(p^A)} \in \mathcal{R}^A$. Since, by Lemma 2, $0 < \pi_m < k\hat{\pi}_i(p_i,N)$, for $\frac{p}{q(p)} > I(N,k)$, then by strict concavity of $k\hat{\pi}_i$, $\exists!\ \frac{\tilde{p}}{q(\tilde{p})} \in \mathcal{R}^B(N)$ with $\frac{\tilde{p}}{q(\tilde{p})} < \frac{p^A}{q(p^A)}$ such that $\pi_m\left(\frac{p^A}{q(p^A)}\right) = k\hat{\pi}_i\left(\frac{\tilde{p}}{q(\tilde{p})}, N\right)$. Still, by strict concavity of $k\hat{\pi}_i(p_i,N)$, $\exists!\ \frac{\tilde{\tilde{p}}}{q(\tilde{\tilde{p}})} > \frac{p^A}{q(p^A)}$ such that $\pi_m\left(\frac{p^A}{q(p^A)}\right) = k\hat{\pi}_i\left(\frac{\tilde{\tilde{p}}}{q(\tilde{\tilde{p}})}, N\right)$. This point is excluded by Assumption 1. Thus, $\forall \frac{p}{q(p)} \in \left[\underline{\mathcal{R}}^B, \frac{\tilde{p}}{q(\tilde{p})}\right)$, the merger is strictly profitable.

case 2: $I(N,k) > \underline{\mathcal{R}}^A$.

For every $\frac{p^A}{q(p^A)} \in \left[I(N,k), \overline{\mathcal{R}}^A\right]$ the same reasoning of case 1 applies. However, for $\frac{p^A}{q(p^A)} \in \left[\underline{\mathcal{R}}^A, I(N,k)\right)$, we have that $\pi_m > k\hat{\pi}_i(p_i,N) > 0$. Thus, $\forall \frac{p^A}{q(p^A)} \in \left[\underline{\mathcal{R}}^A, I(N,k)\right)$, by strict concavity of $k\hat{\pi}_i$, $\exists!\ \frac{\tilde{p}}{q(\tilde{p})} \in \left(\underline{\mathcal{R}}^A, I(n,k)\right)$ such that, $\pi_m\left(\frac{p^A}{q(p^A)}\right) = k\hat{\pi}_i\left(\frac{\tilde{p}}{q(\tilde{p})}, N\right)$. Still, by strict concavity of $k\hat{\pi}_i(p_i,N)$, $\exists!\ \frac{\tilde{\tilde{p}}}{q(\tilde{\tilde{p}})} > \frac{p^A}{q(p^A)}$ such that $\pi_m\left(\frac{p^A}{q(p^A)}\right) = k\hat{\pi}_i\left(\frac{\tilde{\tilde{p}}}{q(\tilde{\tilde{p}})}, N\right)$. This point is excluded by Assumption 1. It follows that, $\forall \frac{p^B}{q(p^B)} \in \left[\frac{p^A}{q(p^A)}, \frac{\tilde{p}}{q(\tilde{p})}\right)$, the merger is strictly profitable. Finally, the merger is also strictly profitable $\forall \frac{p^B}{q(p^B)} \in \left[\underline{\mathcal{R}}^B, \frac{p^A}{q(p^A)}\right]$.

■

Proof of Proposition 2.

Unilateral deviations:

1.1) If an insider leaves the merger, the structure $\{1,2,3,4\}$ emerges. Such deviation is not profitable if:

$$\pi_2^M = \frac{1}{2}\left(\frac{p_2^B q(p_2^B)}{3} - \frac{q(p_2^B)^2}{36}\right) \geq \frac{p_0^A q(p_0^A)}{4} - \frac{q(p_0^A)^2}{32} = \pi_1^{NM} \quad \text{(A-4)}$$

for $\frac{p_0^B}{q(p_0^B)} \in \mathcal{R}^B(4) = \left(\frac{1}{8}, \frac{5}{8}\right)$ and $\frac{p_2^A}{q(p_2^A)} \in \mathcal{R}^A(4,2) = \left(\frac{1}{6}, \frac{1}{3}\right)$. Previous condition is satisfied for either:

$$q(p_0^B) \leq \frac{1}{3}\sqrt{q(p_2^A)(12p_2^A - q(p_2^A))},\ \forall \frac{p_0^B}{q(p_0^B)} \in \mathcal{R}^B(4) = \left(\frac{1}{8}, \frac{5}{8}\right) \quad \text{(A-5)}$$

or:

$$q(p_0^B) > \frac{1}{3}\sqrt{q(p_2^A)(12p_2^A - q(p_2^A))} \text{ and } p_0^B \leq \frac{9q(p_0^B)^2 + 4q(p_2^A)(12p_2^A - q(p_2^A))}{72q(p_0^B)}. \quad \text{(A-6)}$$

and coincides with merger profitability as described in (7).

1.2) If an outsider joins the merger, the structure $\{\mathcal{M}, 4\}$ emerges. Such deviation is not profitable if:

$$\pi_2^{NM} = \frac{p_2^A q(p_2^A)}{3} - \frac{q(p_2^A)^2}{18} \geq \frac{1}{3}\left(\frac{p_3^A q(p_3^A)}{2} - \frac{q(p_3^A)^2}{24}\right) = \pi_3^M \quad \text{(A-7)}$$

for $\frac{p_3^A}{q(p_3^A)} \in \mathcal{R}^A(4,3) = \frac{1}{4}$ and $\frac{p_2^A}{q(p_2^A)} \in \mathcal{R}^A(4,2) = \left(\frac{1}{6}, \frac{1}{3}\right)$. Previous condition is satisfied for:

$$q(p_3^A) \geq \frac{\sqrt{q(p_2^A)(12p_2^A - q(p_2^A))}}{\sqrt{2}}. \quad \text{(A-8)}$$

and coincides with the external stability condition. Both of these conditions are necessary for a CPNE.

Bilateral deviations:

2.1) If two outsiders join the merger, the structure $\{M\}$ emerges. This is excluded by assumption;

2.2) If an insider leaves the merger and an outsider joins it, the market structure stays the same. This deviation is profitable for the insider if $\pi_2^{NM} > \pi_2^M$. On the other hand, it is profitable for the outsider if $\pi_2^M > \pi_2^{NM}$. Clearly, both cannot be satisfied at the same time.

2.3) If both insiders leave the merger, then the structure $\{1,2,3,4\}$ emerges. This deviation is not profitable if the necessary condition (A-6) holds.

Trilateral deviations:

3.1) If an insider leaves the merger and two outsiders join it, the structure $\{\mathcal{M},4\}$ emerges. Such deviation is not profitable for the insider because the condition

$$\pi_3^{NM} = \frac{p_3^A q(p_3^A)}{2} - \frac{q(p_3^A)^2}{8} \leq \frac{1}{2}\left(\frac{p_2^A q(p_2^A)}{3} - \frac{q(p_2^A)^2}{18}\right) = \pi_2^M \tag{A-9}$$

holds for $\frac{p_3^A}{q(p_3^A)} = \frac{1}{4} = \mathcal{R}^A(4,3)$ and $\forall \frac{p_2^A}{q(p_2^A)} \in \mathcal{R}^A(4,2) = \left(\frac{1}{6}, \frac{1}{3}\right)$.

3.2) If two insiders leave the merger and one outsider joins it, the structure $\{1,2,3,4\}$ emerges. This deviation is not profitable for the outsiders if the necessary condition (A-6) holds.

Thus, the market structure $\{\mathcal{M},3,4\}$ is a CPNE if and only if (A-6) and (A-9) hold.

■

Proof of Proposition 3. The result is simply obtained by analyzing the system $\pi_2^M \geq \pi_1^{NM}$, $\pi_2^{NM} \geq \pi_3^M$, and $p_2^A < p_i^B,\ i = \{0,1\}$.

■