Business cycles in a small open economy model: 
The case of Hong Kong.*

Paulina Etxeberria Garaigortá† Amaia Iza‡
The University of the Basque Country.

January, 2011

Abstract

This paper analyzes the business cycle properties of the Hong Kong economy during the 1982-2004 period, which includes the financial crisis experienced in Hong Kong in 1997-98. We show that output, output growth rate and real interest rates volatilities in Hong Kong are higher than their respective average volatilities among developed economies. In this paper, we build a stochastic neoclassical small open economy model that seeks to replicate the main business cycle characteristics of Hong Kong, and through which we try to quantify the role played by exogenous Total Factor Productivity and real interest rates shocks. The main finding is that, in order to explain the Hong Kong business cycle characteristics, the trend volatility has to be higher than the volatility of the transitory fluctuations around the trend, and that the country risk spread is the responsible for both the high variability and its countercyclicality.

Keywords: Asian Financial Crisis, Small Open Economy, Neoclassical model.

JEL Classification: E13, E32, F41.

*We thank Cruz Ángel Echeverría for useful comments. This paper was presented at the All China Economics (ACE) International Conference, Hong Kong, 2006, XXXIII Symposium of Economic Analysis, Spain, 2007, DEGIT XVI Dynamics, Economic Growth and International Trade, Los Angeles, USA, 2009. Financial support from Universidad del País Vasco MACLAB, project IT-241-07, and Ministerio de Ciencia e Innovación, project ECO2009-09732 are gratefully acknowledged. Any remaining errors are the authors’ responsibility.

†Contact details: Departamento de Fundamentos del Análisis Económico II, Avda. Lehendakari Aguirre 83, 48015 Bilbao, Spain. Email: paulina.echeverria@ehu.es

‡Corresponding author: Departamento de Fundamentos del Análisis Económico II Bilbao, Avda. Lehendakari Aguirre, 83, 48015 Bilbao, Spain. Email: amaia.iza@ehu.es. Phone: +34 94 6013785, Fax: 34 94 6017123
1 Introduction

This paper analyzes the business cycle properties of the Hong Kong economy during the 1982Q1-2004Q4 period, which includes the financial crisis experienced in Hong Kong in 1997-98. We compare the cyclical component of Hong Kong with those of other small open economies\(^1\). In particular, we find that output volatility and that of the growth of output in Hong Kong are much higher than their average volatilities among developed economies (2.78 and 5 times higher, respectively). We also concentrate on the relationship between output and real interest rates, as a sharp increase in the interest rates in Hong Kong was observed during the financial crisis. We end by building a stochastic neoclassical small open economy model to try to replicate the main business cycle characteristics of Hong Kong, and through which we try to quantify the role played by exogenous Total Factor Productivity (TFP) and real interest rates shocks in the business cycle characteristics of the Hong Kong economy.

In short, we find that the Hong Kong economy is characterized as an emerging economy due to: \(i\) high output volatility, \(ii\) high volatility of the output growth rate (following Aguiar and Gopinath (2007a, 2007b) we show high volatility of the output growth rate in Hong Kong, which may reveal that shocks to trend rather than shocks to transitory fluctuations around a stable trend may be the primary source of fluctuations), \(iii\) high volatility of real interest rates, \(iv\) high volatility of net exports, \(v\) a negative correlation between real interest rates and output; and, as in small developed economies, by \(vi\) consumption roughly as volatile as output, and \(vii\) a negative correlation between net exports and output similar to small developed economies.

The business cycle characteristics of the Hong Kong economy have been previously studied by Leung and Suen (2001), whose work does not include the financial crisis and post-crisis period. They analyze some aspects of the business cycle characteristics of the Hong Kong economy between 1964 and 1994 for annual data and quarterly data (1976Q1-1994Q1), using the BP filter. These authors find that the volatility of output in the 1979Q1-1996Q4 period was around 2.3%, which is higher than the standard deviation of quarterly output in the US (1.7%). They also conclude that there is little consumption smoothing over the business cycle since the relative volatility of consumption with respect to output is close to 1.0.

\(^1\)Hong Kong is classified as a developed or emerging economy by different organizations: IFS, JPMorgan, the United Nations and the Economist Intelligence Unit count Hong Kong as an “emerging economy”. On the other hand, the World Economic Outlook and the World Equity Index Group from Salomon Smith Barney categorize it as an “advanced economy”. Morgan Stanley Capital International leaps Hong Kong in its developed-market index. And it is recently identified as a newly industrialized country by organizations such as Standard and Poor (2000) and the International Finance Corporation.
(but lower than) one. They remark that the contractions in Hong Kong seem to be “steeper” than expansions and there is a marked seasonal pattern in the economic series. Moreover, downturns seem to be triggered by external events in Hong Kong. Finally Leung and Suen (2001) find a positive contemporaneous correlation between real interest rate (best lending rate minus CPI inflation) and GDP.

Crosby (2004) also analyzes the time series properties of output, among other variables (government expenditure, exports, imports, CPI, real exchange rate), in Hong Kong, for the 1974Q3-1999Q4 period. He also applies the BP filter to detrend the series and finds that the output volatility is around 3%.

With regard to the real interest rate, we first observe a sharp increase of interest rates during the Asian financial crisis. As already noted by several authors, such as Neumeyer and Perri (2005), Oviedo (2005), Uribe and Yue (2006) among others, real interest rates during economic expansion appear to be low, while periods of economic stress are often accompanied by high real interest rates. Edwards and Susmel (2000) use univariate SWARCH models to analyze the interest rate volatility in five countries that have undergone a financial crisis: Chile, Argentina, Mexico, Brazil and Hong Kong. They conclude that in Hong Kong, unlike in other countries such as in Mexico or Brazil, the economy shifted to a high volatility interest rate state during the financial crisis, and stayed there for almost a year. In particular, interest rate volatility in Hong Kong in the 1982-2004 period was 0.68%, whereas it was only 0.21% in the US. The second interesting finding is that, as shown in Neumeyer and Perri (2005) for other small open emerging countries, real interest rates in Hong Kong are countercyclical (−0.32), similar to small emerging economies, and there is a clear pattern of leading the cycle.

The main focus of this paper is to assess the quantitative impact of exogenous shocks (interest rates shocks and TFP shocks) on the characteristics of the Hong Kong business cycles and to understand the channels through which they work. For that purpose, we develop a stochastic neoclassical small open economy model that partly follows the model built by Neumeyer and Perri (2005) and partly follows the model built by Aguiar and Gopinath (2007a). The model consists of an infinitively lived representative household, a production sector formed by an indeterminate number of competitive firms producing with the same constant returns to scale production technology, using capital and labor as inputs. As we are considering a small open economy, the foreign debt, or international bond, yields

---

2Neumeyer and Perri (2005) and Uribe and Yue (2006) stress that the real interest rates on external bonds show a negative correlation with the cycle in emerging markets. Kaminsky et al. (2004) also show that domestic short-term treasury rates deflated by consumer price inflation in those countries are countercyclical.
a real rate of return that is viewed as exogenous by the agents in the economy. As in Aguiar and Gopinath (2007a), we decompose the shocks of the Solow residuals into two components: a transitory productivity shock around the trend and a shock to the trend. The structure of the model not only follows that of Neumeyer and Perri (2005) and Aguiar and Gopinath (2007a), but it is also close to the small open economy models studied by Mendoza (1991) or Oviedo (2005b).

As in Neumeyer and Perri (2005), we will consider the same simple theory behind the high volatility of the interest rates. We will consider that the volatility of the interest rate, at which the Hong Kong economy can borrow from international lenders, depends upon two components. On the one hand, it will be affected by the volatility of the international rate for risky assets and, on the other hand, it will depend on the country spread over the international rate.

The remainder of the paper is organized as follows. Section 2 briefly describes some characteristics of the Hong Kong business cycles and compares them to other small open developed and emerging economies. In Section 3, we develop the model. In section 4 we calibrate it. In section 5 we obtain the main findings. Section 6 summarizes and concludes.

2 Business cycles characteristics of the Hong Kong economy.

This section analyzes the business cycle properties of the Hong Kong economy, with special emphasis on the real interest rate and output\(^3\). In particular, we use data obtained from Hong Kong Census and Statistics Department\(^4\) and consider the period that includes the Asian financial crisis (1982Q1-2004Q4). We show that the economy of Hong Kong shares some characteristics with a standard emerging market economy and others with a small open developed economy. We compare the cyclical components of Hong Kong with those of other small countries such as Korea, Philippines, Thailand, Netherlands, Norway and

\(^3\)An alternative way of characterizing Hong Kong business cycles is developed by Calderon and Fuentes (2010) following Harding and Pagan (2002). They identify turning points in the (log) level of GDP.

\(^4\)Two other different databases have also been used: \(i\) we use data from the International Financial Statistics (IFS) in order to obtain the most accurate and internationally comparable results and \(ii\) data from the Penn World Table for the 1979-2000 period for consumption, government expenditure, investment and output is also used. We obtain very similar results using data from IFS and Hong Kong census. However, we find that the volatility of the series from the Penn World Table are lower than those obtained from the Hong Kong statistics data and IFS when the corresponding series have been detrended either by HP or BP filters.
Sweden. Data for other small open economies is from Aguiar and Gopinath (2004) for a very similar period. The sample consists of data from at least 40 quarters and all series have been detrended using the BP filter. The comparison is in terms of relative volatility and correlations as performed in Uribe (2007) and Aguiar and Gopinath (2004).

Neumeyer and Perri (2005), Aguiar and Gopinath (2004) or Uribe (2007) compare business cycle characteristics of small open developed and emerging economies (13 developed and 13 emerging, not including Hong Kong). In order to compare the business cycles from Hong Kong to other small open economies, we apply the BP filter at frequencies between 6 and 32 quarters with 12 leads and lags to the data series from the countries that Aguiar and Gopinath analyze. Averages for emerging and developed small open economies are from Aguiar and Gopinath (2004).

In the case of Hong Kong, the filter selection is not indifferent to the obtained results. Time series analyses that characterize for containing high frequency components show a remarkable difference in the cyclical components obtained from the de-trending of HP and BP filters. The high-frequency components pass through the HP filter, but are removed by the BP filter. Hong Kong is an economy with high frequency components. Figure 1 shows the cyclical component of the quarterly series of the output in Hong Kong, which have been detrended using the HP and BP filters, respectively. We see that the HP filtered series are less smooth and that there is a marked seasonal pattern in the output of the economy, in particular during the nineties. When looking at HP filtered data, we see the existence of high frequency in the economy. The behavior of the cyclical component when the BP filter is used is similar to the one obtained by Leung and Suen (2001).

---

5 More data description is in Appendix A.

6 The countries that these authors analyze are: 13 small emerging countries: Argentina, Brazil, Ecuador, Israel, Korea, Mexico, Peru, Philippines, Slovak Republic, South Africa, Thailand, and Turkey; 13 small developed countries: Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden and Switzerland.

7 Rand and Tarp (2002) study some characteristics of business cycles for 50 developing countries and they argue that the cycles in these countries are shorter (between 7 and 18 quarters) than in advanced economies, making it necessary to modify the filtering procedures normally applied for industrialized countries. Therefore, according to these authors, relying on the smoothing parameters always used for developed countries when studying poor countries is “at best ad hoc, and may lead to inappropriate conclusions as regards the summary statistics (or stylized facts) that characterize macroeconomic fluctuations. In the extreme, inappropriate numerical models might be validated and vice versa, depending on the choice of smoothing parameter”. They also find that the “data analysis shows that a downward adjustment in the standard deviation of the Asian countries that they study, when using the BP-filter instead of the HP-filter, is more pronounced in Asian countries than in North African countries”. Therefore, these results highlight the importance of using appropriate filters before drawing any final conclusions about business cycle properties of the economy of Hong Kong.
Therefore, it seems important to eliminate the cycles that last less than 6 quarters. The series would otherwise be too seasonal. We therefore filtered each series using the BP filter at frequencies between 6 and 32 quarters with 12 leads and lags (this means that three years of data are lost both at the beginning and at the end of the sample).

Figure 1: Cyclical component of GDP, 1973Q1-2005Q4.

Emerging economies, according to Aguiar and Gopinath (2007) and Neumeyer and Perri (2005) are characterized by, on average, i) a business cycle twice as volatile as that of their developed counterparts; ii) also the output growth rate is twice as volatile as that of the developed economies; iii) output displays roughly the same autocorrelation as that of developed economies; iv) consumption is roughly 40 percent more volatile than income in emerging economies (in developed economies the ratio is slightly less than 1 on average); v) trade balance-to-output is much more countercyclical in emerging countries than in developed countries (around $-0.5/-0.8$ in emerging economies, $-0.17$ in developed economies); vi) interest rates are countercyclical and lead the cycle in emerging economies while they are acyclical and lag the cycle in developed economies.

Table 1 reports key moments for the Hong Kong economy, for the 1982Q1-2004Q4 period, and those moments calculated by Aguiar and Gopinath (2004) of the business cycle averaged over 13 emerging and 13 developed small open economies. Table 2 compares the key moments of business cycles in Hong Kong for the 1982Q1-2004Q4 period with those from other small
open emerging and developed economies for a similar time period.

Table 1: Emerging vs. Developed Economies (Averages) and Hong Kong

<table>
<thead>
<tr>
<th></th>
<th>Emerging Markets</th>
<th>Developed Markets</th>
<th>Hong Kong</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(Y)$</td>
<td>2.02</td>
<td>1.04</td>
<td>2.89</td>
</tr>
<tr>
<td>$\sigma(\Delta Y)$</td>
<td>1.87</td>
<td>0.95</td>
<td>4.76</td>
</tr>
<tr>
<td>$\sigma(C)/\sigma(Y)$</td>
<td>1.32</td>
<td>0.94</td>
<td>0.84</td>
</tr>
<tr>
<td>$\sigma(I)/\sigma(Y)$</td>
<td>3.96</td>
<td>3.42</td>
<td>3.40</td>
</tr>
<tr>
<td>$\sigma(G)/\sigma(Y)$</td>
<td>-</td>
<td>-</td>
<td>0.69</td>
</tr>
<tr>
<td>$\sigma(NX/Y)$</td>
<td>2.09</td>
<td>0.71</td>
<td>2.15</td>
</tr>
<tr>
<td>$\rho(NX/Y,Y)$</td>
<td>-0.58</td>
<td>-0.26</td>
<td>-0.33</td>
</tr>
<tr>
<td>$\rho(C,Y)$</td>
<td>0.74</td>
<td>0.69</td>
<td>0.79</td>
</tr>
<tr>
<td>$\rho(I,Y)$</td>
<td>0.87</td>
<td>0.75</td>
<td>0.87</td>
</tr>
<tr>
<td>$\rho(G,Y)$</td>
<td>-</td>
<td>-</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Note: All series are filtered using BP filter at frequencies between 6 and 32 with 12 leads and lags, except first differences in output. All series are logged, except net exports. Hong Kong statistics are based on quarterly data for the period 1982Q1-2004Q4. Data for emerging and developed economies comes from Aguiar and Gopinath (2004). Standard deviations (volatilities) are in percentages. The notation $\rho(A,B)$ denotes the correlation coefficient between the variable A and the variable B.

Figure 2 presents Hong Kong GDP and its components: consumption, investment, government purchases and the trade balance, all detrended using the BP filter.

2.1 Real Gross Domestic Product

We obtain that the volatility of Real Gross Domestic Product is 2.48%, twice as volatile as in developed small open economies and similar to emerging economies. If we take a shorter series (1982Q1-1997Q3), just prior to the crisis, we obtain (approximately) the same result (2.69%). The inclusion of the crisis period does not modify the Hong Kong output volatility.

We also look at first differences in unfiltered log GDP as recent literature (Aguiar and Gopinath (2007a), Boz et al. (2008), Schmitt-Grohé and Uribe (2008)) distinguishes between permanent and transitory components of TFP shocks, in order to analyze business cycles of emerging economies. The Aguiar and Gopinath (2007a) hypothesis is that emerging markets are characterized by a volatile trend that determines the behavior of the economy.
Table 2: Business Cycles in Hong Kong and in several emerging and developed economies

<table>
<thead>
<tr>
<th></th>
<th>(\sigma(Y))</th>
<th>(\sigma(\Delta Y))</th>
<th>(\sigma(C)/\sigma(Y))</th>
<th>(\sigma(I)/\sigma(Y))</th>
<th>(\sigma(NX/Y))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hong Kong</strong></td>
<td>2.89</td>
<td>4.76</td>
<td>0.84</td>
<td>3.40</td>
<td>2.15</td>
</tr>
<tr>
<td><strong>Emerging economies</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Korea</td>
<td>2.57</td>
<td>1.71</td>
<td>1.23</td>
<td>2.45</td>
<td>2.48</td>
</tr>
<tr>
<td>Philippines</td>
<td>2.58</td>
<td>1.66</td>
<td>0.51</td>
<td>4.47</td>
<td>2.21</td>
</tr>
<tr>
<td>Thailand</td>
<td>5.56</td>
<td>2.25</td>
<td>1.25</td>
<td>3.65</td>
<td>6.25</td>
</tr>
<tr>
<td><strong>Developed economies</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.01</td>
<td>0.88</td>
<td>1</td>
<td>2.38</td>
<td>0.52</td>
</tr>
<tr>
<td>Norway</td>
<td>1.18</td>
<td>1.46</td>
<td>1.57</td>
<td>4.24</td>
<td>1.45</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.22</td>
<td>1.45</td>
<td>1.05</td>
<td>4.46</td>
<td>0.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(\rho(C,Y))</th>
<th>(\rho(I,Y))</th>
<th>(\rho(NX/Y,Y))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hong Kong</strong></td>
<td>0.79</td>
<td>0.87</td>
<td>-0.33</td>
</tr>
<tr>
<td><strong>Emerging economies</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Korea</td>
<td>0.89</td>
<td>0.84</td>
<td>-0.70</td>
</tr>
<tr>
<td>Philippines</td>
<td>0.62</td>
<td>0.86</td>
<td>-0.60</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.98</td>
<td>0.99</td>
<td>-0.95</td>
</tr>
<tr>
<td><strong>Developed economies</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.73</td>
<td>0.77</td>
<td>-0.29</td>
</tr>
<tr>
<td>Norway</td>
<td>0.77</td>
<td>0.14</td>
<td>-0.08</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.13</td>
<td>0.79</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Note: All series are filtered using BP filter at frequencies between 6 and 32 with 12 leads and lags, except first differences in output. All series are logged, except net exports. Hong Kong statistics are based on quarterly data for the 1982Q1-2004Q4 period. Data for emerging and developed economies comes from Aguiar and Gopinath (2004). Standard deviations (volatilities) are in percentages. The notation \(\rho(A,B)\) denotes the correlation coefficient between the variable A and the variable B.

at business cycle frequencies. According to these authors, shocks to trend, rather than transitory fluctuations around a stable trend, are the primary source of fluctuations in emerging markets. In Hong Kong, the volatility of unfiltered first differences in log GDP is 4.76%, much higher than the average values from emerging economies. Therefore, the possibility could be considered that the permanent component of productivity shocks plays a main role in the Hong Kong economy and that transitory shocks are less important.
2.2 Consumption

The relative volatility of consumption to output in developing countries is around 40% larger than in industrial countries. Neumeyer and Perri (2005) or Aguiar and Gopinath (2007a) find that consumption tends to be more volatile than output in emerging economies while it is roughly as volatile as output in developed economies. Average consumption volatility over GDP in small open emerging economies is 1.45, while the value falls to 0.94 for small open developed economies\(^8\).

In Hong Kong, the relative volatility of consumption with respect to GDP is 0.84 for the 1982Q1-2004Q4 period. This result is also found in Leung and Suen (2001). Compared to other small economies, the obtained result is similar to small open economies such as Korea or Sweden and Norway. Nevertheless, this result is not independent of the period analyzed. If we take the 1973Q1-2005Q4 time period, the consumption volatility relative

\(^8\)Following a different methodology, Rand and Tarp (2002) also find that both total and private consumption are generally more volatile than output in their sample of 15 Latin American countries and in 20 Sub-Saharan African countries.
to GDP increases to 1.07. It seems that in Hong Kong it is easier to smooth consumption during the last decade than it was during the 1970s. Correlation between consumption and GDP is positive and high, 0.79, closer to the average for small emerging economies.

2.3 Government expenditure

Here we also look at the Government expenditures. The relative volatility of government expenditures to GDP is 0.69 for the 1982Q1-2004Q4 period. This relative volatility slightly increases when we exclude the period after the Asian financial crisis (1998Q1-2004Q4). During the 1982Q1-1997Q4 period, the relative volatility of government purchases to output is slightly higher, 0.71.

There is a positive correlation between government consumption and GDP (0.15), for the 1982Q1-2004Q4 period. If we consider the period prior to the financial crisis (1982Q1-1997Q4), we find that government expenditure is acyclical. Thus, it was procyclical during the crisis and as can be observed in Figure 2.

We also obtain that the correlation between government consumption and net exports is $-0.43$. This negative correlation is even greater if we exclude the period after the crisis from the analysis. It appears that government expenditure and net exports during the 1998Q1-2004Q4 period behaved in a more procyclical way than in the 1982Q1-1997Q4 period.

2.4 Investment

The relative volatility of investment in Hong Kong is around 3.4 times the volatility of gross domestic product, a value that matches well within the range has been found in both emerging and developed countries. This result does not depend on the period considered.\(^9\)

We also find a strongly positive contemporaneous correlation, 0.87, between de-trended investment and GDP data in Hong Kong. The correlation between investment and output peaks with a lag of one, even though it is practically the same at time zero and with a lag of one quarter (0.873 at time zero and 0.879 at a lag of one). In the US economy, the correlation between investment and output has a peak at time zero and in European countries peaks with a lag of one.

\(^9\)In Leung and Suen (2001) investment is divided into two components: investment in fixed capital and change in inventories. Only the volatility of change in inventories is more than double GDP volatility.
2.5 Trade

A striking feature of emerging market economies is the strongly countercyclical nature of the trade balance-to-GDP compared to developed economies. On average, emerging markets have a correlation of net exports to output with GDP of $-0.58$ and developed ones have a correlation of $-0.26$.

In Hong Kong, the correlation between net exports over GDP and GDP is $-0.33$. This countercyclical behavior was more pronounced during the period after the Asian financial crisis, 1998Q1-2004Q4. When the period immediately prior to the crisis is considered (1982Q1-1997Q4), the countercyclical value halves to $-0.16$.

With regard to the volatility of net exports-to-GDP (2.15%), it is very similar to the average for emerging economies (around 2.09%).

We also analyze both net export components separately. With regard to the cyclical component of both imports and exports, we obtain a relative volatility of around twice the volatility of GDP. This result does not depend on the time horizon considered and is also obtained by Leung and Suen (2001). The relative volatility of imports and exports in Hong Kong is lower than that for the US (Stock and Watson (1999)).

Regarding the correlations between these variables and GDP, we find that imports and exports are significantly pro-cyclical, which characterizes the Hong Kong economy as a small open economy. In particular, the correlation between exports and gross domestic product in Hong Kong is 0.77, which is clearly much higher than that observed in the US (0.2) (Stock and Watson (1999)). The correlation between imports and output is 0.79.

As mentioned before, we also looked at correlation between net exports and government expenditure in Hong Kong. We find a value of 0.43 for the 1982Q1-2004Q4 period.

2.6 Real interest rates

We finish this section by analyzing the cyclical component of the real interest rate (the three-month interest rate from the Hong Kong Monetary Authority (HKMA) dataset\textsuperscript{10}) for the 1982Q1-2004Q4 period, and, over all, its relationship with the cyclical component of the GDP in Hong Kong. The gross real interest rate is obtained by dividing gross nominal interest rate by the expected gross inflation rate. The expected gross inflation rate is computed as

\textsuperscript{10}Other interest rate measures were considered such as the Hong Kong Best Lending rate from HKMA for period 1082Q1-2004Q4 or the Three-month Treasury Bill from IFS dataset for the 1993Q4-2007Q2 period. Results are similar.
the average for the current quarter prices and the value of the GDP deflator inflation in the three preceding quarters\textsuperscript{11}. More details in Appendix A.

Concerning the comparison of the cyclical component of the real interest rate and the cyclical component of the GDP, we carry out an analysis as performed by Neumeyer and Perri (2005).

Figure 3 shows the observed evolution of the Hong Kong gross real interest rate compared to the US gross real interest rate (the real three-month Treasury bill) for the 1982Q1-2004Q4 period. It is noteworthy that the US interest rate is much more stable than the Hong Kong interest rate during the whole period. In particular, the Hong Kong interest rate increased dramatically during the Asian Financial crisis while the US interest rate remained quite stable. The Hong Kong nominal three-month interest rate leapt 1.5 percentage points on average between 1997 and 1998 (2 percentage points between 1997Q3 and 1997Q4, then decreased by 2 percentage points in the next quarter and increased by 3 percentage points between 1998Q1 and 1998Q2). The Hong Kong real interest rate leapt 1.36% (1.5% the nominal interest rate) on average between 1997 and 1998. The US interest rate remained almost constant during the same period. Therefore, most of the rise comes from the spread in Hong Kong interest rates over world (US) interest rates. It could reflect the sudden decline in the willingness to lend to Hong Kong firms.

Figure 4 documents the cyclical components of real GDP and real interest rate for Hong Kong. As can be seen graphically, there is a negative comovement between GDP and real interest rates. After applying the BP filter to log(GDP) and real interest rate, we find a negative correlation of $-0.32\quad(-0.39$ if the financial and post-financial period is excluded)\textsuperscript{12}. On the other hand, the correlation between US interest rate and Hong Kong output is slightly negative, $-0.15$, and the correlation between the country risk spread from Hong Kong and its output is $-0.32$. Correlation between Hong Kong interest rate and US interest rate is 0.56, and the correlation between Hong Kong interest rate and Hong Kong country spread is 0.95.

Table 3 shows the volatility and correlations of the Hong Kong interest rate, the

\textsuperscript{11}A four period lag to compute expected inflation has also been analyzed but the results do not vary.

\textsuperscript{12}A negative correlation is also found when the real Best Lending interest rate is used for the 1980-2003 period. On the other hand, Leung and Suen (2001) report a positive correlation between the Real Best Lending interest rate and output, 0.32, for the 1977Q4-1994Q2 period. They calculate it as the nominal best lending rate minus CPI inflation. We calculate the three-month interest rate deflated with GDP deflator. Contrary to their results, we obtain a negative correlation between output and real interest rate for both periods that include and exclude the Asian financial crisis. Dotsey et al. (2003) show that results can vary depending on the deflator for interest rate used.
international interest rate (US interest rate) and the country risk premium with output, consumption, investment and exports for the period before the crisis (1982Q1-1997Q4) and a longer period that includes the Asian financial crisis (1982Q1-2004Q4). The volatility of the Hong Kong real interest rate is 0.68%, more than three times the US real interest rate volatility 0.21%.

Figure 5 shows the cross-correlation between output and real interest rate for Hong Kong at different leads and lags. It can be seen that the Hong Kong real interest rate is countercyclical and that the real interest rate in Hong Kong leads the cycle by 3 quarters\(^\text{13}\). Cross-correlation of real interest rate \((t + k)\) with output \(t\) exhibits a U-shape pattern and leads the cycle by 3 quarters.

Table 4 shows the volatility and correlation between real interest rates and output for Hong Kong and other small open economies (Korea, Philippines, Netherlands, Sweden)\(^\text{14}\). If we compare Hong Kong interest rates to other economies, we find that the volatility of the real

\(^{13}\)According to Zimmerman (2005), a variable leads (lags) the cycle when its correlation with output (leadded or lagged) takes maximum value.

\(^{14}\)The obtained correlations between the interest rate and output are the same with IFS and HKMA data when the same period is considered. Moreover, the results are very similar when the expected inflation is calculated using four lags instead of using three lags and the contemporaneous inflation rate.
interest rate in Hong Kong (0.68%) is higher than in the other economies (the next higher is Sweden (0.39%)). We also find that correlation of consumption with interest rate is negative (−0.22), correlation between interest rate and investment is negative (−0.11), correlation between government expenditure and interest rate is positive (0.09) and correlation between net export over GDP and real interest rate is negative (−0.13).

As we have mentioned, Neumeyer and Perri (2005) find that real interest rates in small open emerging economies are countercyclical and lead the cycle, while real interest rates in developed economies are acyclical and lag the cycle. They find that there is a negative co-movement between output and real interest rate in emerging economies (real interest rates are countercyclical), while there is no such pattern in developed economies (real interest rates are acyclical or mildly procyclical depending on the countries). They also find that real interest rates in the five analyzed emerging economies lead the cycle by one quarter, while real interest rates in the developed economies, on average, lag the cycle by three quarters. Uribe and Yue (2003) likewise find a strong negative correlation between real interest rates and economic activity in a sample of seven emerging markets.

There are several studies (Neumeyer and Perri (2005), Uribe and Yue (2006), Otsu (2008)) that analyze the effects of real interest rate shocks on emerging market business
cycles based on the countercyclical behavior of real interest rates in emerging economies. In the next section we will develop a small open economy model with interest rate shocks, and a distinction between the transitory and permanent shocks to the TFP. According to Otsu (2010), the inclusion of interest rate shocks should help to understand the drop in consumption during the Asian Financial crisis, particularly in Hong Kong.
Figure 5: Cross-correlations between output and interest rate.

Table 4: Interest rates. Volatility and Correlations.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma(R)$</th>
<th>$\rho(Y, R)$</th>
<th>$\rho(C, R)$</th>
<th>$\rho(G, R)$</th>
<th>$\rho(I, R)$</th>
<th>$\rho(NX/R, R)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hong Kong</td>
<td>0.68</td>
<td>-0.32</td>
<td>-0.22</td>
<td>0.09</td>
<td>-0.11</td>
<td>-0.13</td>
</tr>
<tr>
<td>Korea</td>
<td>0.26</td>
<td>-0.98</td>
<td>-0.98</td>
<td>-0.36</td>
<td>-0.93</td>
<td>0.96</td>
</tr>
<tr>
<td>Philippines</td>
<td>0.17</td>
<td>-0.57</td>
<td>-0.64</td>
<td>-0.57</td>
<td>-0.50</td>
<td>0.39</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.19</td>
<td>0.33</td>
<td>0.55</td>
<td>0.13</td>
<td>0.50</td>
<td>-0.48</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.39</td>
<td>-0.11</td>
<td>-0.25</td>
<td>0.18</td>
<td>0.003</td>
<td>-0.35</td>
</tr>
</tbody>
</table>

Note: All series are logged, except interest rates and net exports, and BP filtered. Hong Kong data is 1982Q1-2004Q4. Standard deviations (volatilities) are in percentages. The notation $\rho(A, B)$ denotes the correlation coefficient between the variable A and the variable B.
3 The economic model

3.1 Description of the basic model

This section describes the economic model, which is based on Neumeyer and Perri (2005) and on Aguiar and Gopinath (2007a). As we have observed that the volatility of the first log differences of GDP is quite high in Hong Kong, we have distinguished, as in Aguiar and Gopinath (2007a, 2007b), the transitory and permanent shocks to the Total Factor Productivity. The model is a small open economy neoclassical model. The economy consists of identical infinitely-lived households, an indeterminate number of competitive firms producing the same single good using the same constant returns to scale technology. Households can trade a single asset in international financial markets. Firms also trade in the asset because of the assumption of working capital requirements. There are adjustment costs on both capital stock and international debt. There are external shocks to the economy that affect the real interest rate and the total factor productivity of the Hong Kong economy. Time is discrete.

3.1.1 Households

Households are represented by a continuum of infinitely-lived, identical households of measure one. The representative household chooses how much to work, consume, invest and borrow, such that maximizes its lifetime utility function,

\[ U = E_0 \sum_{t=0}^{\infty} \beta^t u (c_t, l_t) , \]  

where the functional form of the utility function is GHH preferences:

\[ u (c_t, l_t) = \frac{1}{1 - \sigma} \left[ c_t - \psi \Gamma_t l_t^v \right]^{1-\sigma} , \]  

where \( c_t \) denotes household consumption at time \( t \), \( l_t \) denotes household labor at time \( t \). The parameter \( \sigma \) is the curvature of the period utility, \( \psi > 0 \), and \( v \) determines the wage elasticity of labor supply, which is given by \( 1/(v - 1) \), with \( v > 1 \), in order to ensure that the elasticity of labor supply is positive. The symbol \( \Gamma_t = \gamma_t \Gamma_{t-1} \) represents the cumulative product of stochastic growth rates of labor-augmenting technological progress up to time \( t \), which will be discussed below. The growth rate of the Total Factor Productivity at each period \( t \) will be given by \( (\gamma_t - 1) \).

The GHH preferences were first proposed by Greenwood, Hercovich and Huffman.
(1988) and have the special property that labor supply depends only on the current wage and is independent of consumption or income (there are no wealth effects on the labor supply decision). In a small open economy, this type of preferences better captures the relative volatility of consumption with respect to output (see Correia, Neves and Rebelo (1995)). A well-known fact in the small open economy RBC literature is that with Cobb-Douglas preferences over consumption and leisure, the model predicts too much consumption smoothing. GHH preferences are introduced in the model to increase consumption volatility. Also, as noted in Fernández-Villaverde et al. (2010), these preferences are able of generating a contraction in consumption, labor and output after a positive shock to the interest rate level.

In order for these GHH preferences to be consistent with a balanced growth path, the disutility of work in the market has to increase with the cumulative growth rate of labor-augmenting technological progress $\Gamma_{t-1}$. It has to be assumed that $\beta\gamma_{ss}^{1-\sigma} < 1$ to ensure that utility is finite. As we will see, this condition coincides with the transversality condition and the No-Ponzi game conditions that should also be satisfied.

Assuming, as in Neumeyer and Perri (2005), that at the time households make their investment decisions, they know the rate of return of international real bonds (see also Uribe and Yue (2006), Fernández-Villaverde et al. (2010)), but they do not know the rates of return of domestic physical capital, the household budget constraint at time $t$ can be written as follows:

$$c_t + i_t + b_t + \kappa(b_t) = w_t l_t + r_t k_{t-1} + R_{t-1} b_{t-1},$$

where the subscript $t$ denotes that for the corresponding variable, the history of shocks up to those which have been revealed at the beginning of time $t$, is known. Therefore, for example, $b_{t-1}$ means that the amount of foreign bonds that households have at the beginning of period $t$ depends on the history of shocks revealed up to the beginning of period $t - 1$. Shocks revealed at the beginning of period $t$ do not affect the real interest rates on the international bonds held by households at time $t$, $R_{t-1}$. Households spend their labor income, $w_t l_t$, and capital income, $r_t k_{t-1}$, and the returns from foreign asset holdings, $R_{t-1} b_{t-1}$, on consumption, $c_t$, investment, $i_t$, foreign bonds, $b_t$, and the cost of holding foreign bonds, $\kappa(b_t)$, which are included in order to avoid a unit root for the foreign bonds (see, among others, Neumeyer and Perri (2005)). We assume that the bond holding cost function is $\kappa(b_t) = \left[\frac{\chi}{2} y_t \left(\frac{b_t}{y_t} - \overline{b} \right)^2\right]$, i.e., the cost of holding bonds are applied to the amount of bonds that households buy at
period $t$.

The law of motion for the physical capital is:

$$k_t = i_t + (1 - \delta)k_{t-1} - \phi(k_{t-1}, k_t), \quad (4)$$

where $0 < \delta < 1$ is the rate of capital depreciation and $\phi(k_{t-1}, k_t)$ denotes the adjustment costs. We assume the same adjustment cost function of capital as in Neumeyer and Perri (2005),

$$\phi(k_{t-1}, k_t) = \varphi \frac{k_t}{k_{t-1}} \left( \frac{k_t}{k_{t-1}} - \gamma_{ss} \right)^2, \quad (5)$$

where $\varphi > 0$.

Finally, we also assume that household borrowing is bounded below so as to rule out Ponzi schemes, therefore:

$$\lim_{j \to \infty} E_t \frac{b_{t+j}}{\prod_{s=0}^{j} R_{t+s-1}} \geq 0, \quad (6)$$

$$\lim_{j \to \infty} E_t \frac{k_{t+j}}{\prod_{s=0}^{j} R_{t+s-1}} \geq 0. \quad (7)$$

Hence, the household’s problem is to choose the sequence of consumption, labor, investment in physical capital and foreign asset holdings such that maximize the expected utility (1) subject to the budget constraint (3), the capital accumulation constraint (4) and the No-Ponzi game conditions (6) and (7), taking as exogenously given a sequence of prices of labor, capital and of foreign assets, $\{w_t, r_t, R_{t-1}\}_{t=0}^{\infty}$ and for given values of initial foreign assets, $b_{-1}$, and physical capital, $k_{-1}$.

$$-\frac{U_{l_t}}{U_{c_t}} = w_t, \quad (8)$$

$$U_{ct} \left[ 1 + \varphi \left( \frac{k_t}{k_{t-1}} - \gamma_{ss} \right) \right] = \beta E_t U_{ct+1} \left[ 1 - \delta + r_{t+1} + \frac{\varphi}{2} \left( \frac{k_{t+1}}{k_t} \right)^2 - \gamma_{ss}^2 \right], \quad (9)$$

---

15 In this way, the adjustment costs of international bonds can be decentralized as in Uribe and Yue (2006), through financial intermediation by domestic banks. Then, the shadow interest rate faced by domestic agents will take into account the operational costs domestic banks face (see page 19 in Uribe and Yue (2006)).
\[ U_{ct} \left[ 1 + \chi \left( \frac{b_t}{y_t} - \bar{b} \right) \right] = \beta R_tE_tU_{ct+1}. \] (10)

and the budget constraint at each period:

\[ c_t + k_t - (1 - \delta)k_{t-1} + \frac{\varphi}{2}k_{t-1}\left( \frac{k_t}{k_{t-1}} - \gamma_{ss} \right)^2 + \ldots \] (11)

\[ b_t + \frac{\chi}{2}y_t \left( \frac{b_t}{y_t} - \bar{b} \right)^2 = w_t l_t + r_t k_{t-1} + \frac{R_{t-1} b_{t-1}}{y_t}. \]

These equations show that \textit{i)} the marginal rate of substitution of leisure for consumption has to be equal to their relative price, \textit{ii)} the Euler equation for capital, \textit{iii)} the Euler equation for foreign bonds and \textit{iv)} that the household budget constraint must be satisfied. Moreover, the transversality conditions for physical capital and foreign assets must hold:

\[ \lim_{t \to \infty} \beta_t^t U_{ct} k_t = 0, \] (12)

\[ \lim_{t \to \infty} \beta_t^t U_{ct} b_t = 0. \] (13)

With specific GHH preferences we obtain the following (14), (15) and (16) equations:

\[ \psi v \Gamma_{t-1} l_t^{(v-1)} = w_t, \] (14)

\[ (c_t - \psi \Gamma_{t-1} l_t^{v})^{-\sigma} \left[ 1 + \varphi \left( \frac{k_t}{k_{t-1}} - \gamma_{ss} \right) \right] = \beta E_t(c_{t+1} - \psi \Gamma_{t+1} l_{t+1}^{v})^{-\sigma} \left[ 1 - \delta + r_{t+1} + \frac{\varphi}{2} \left( \left( \frac{k_{t+1}}{k_t} \right)^2 - \gamma_{ss}^2 \right) \right], \] (15)

\[ (c_t - \psi \Gamma_{t-1} l_t^{v})^{-\sigma} \left[ 1 + \chi \left( \frac{b_t}{y_t} - \bar{b} \right) \right] = \beta R_tE_t(c_{t+1} - \psi \Gamma_{t+1} l_{t+1}^{v})^{-\sigma}. \] (16)

### 3.1.2 The production sector

In this economy, the production sector is formed by an indeterminate number of identical competitive firms. Firms produce a tradable good, whose spot price is normalized to unity without loss of generality, according to the following Cobb-Douglas production function
\begin{equation}
y_t = z_t k_{t-1}^\alpha \left[ \Gamma_t l_t \right]^{1-\alpha},
\end{equation}

where \( y_t \) is final output that is realized at the end of period \( t \), \( k_{t-1} \) is the stock of physical capital and \( l_t \) is labor, both hired at the beginning of period \( t \), \( z_t \) is the stochastic variable that will characterize the transitory component of the total factor productivity (whose value is known at the beginning of period \( t \)), and \( \Gamma_t \) denotes the cumulative stochastic growth rate of labor-augmenting technological change. The transitory component of the Total Factor Productivity, \( z_t \), will be characterized, as usual, as an AR (1) process:

\begin{equation}
\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{z,t},
\end{equation}

where \(|\rho_z| < 1\) and \( \varepsilon_{z,t} \) is a zero mean, i.i.d. process, drawn from a Normal distribution, with \( \text{Var}(\varepsilon_{z,t}) = \sigma_z^2 \).

The law of motion for \( \Gamma_t \) is as follows:

\begin{equation}
\Gamma_t = \gamma_t \Gamma_{t-1},
\end{equation}

where \((\gamma_t - 1)\) denotes the shock to the growth rate of the Total Factor Productivity, i.e., the permanent shock to the Total Factor Productivity. We assume, as in Aguiar and Gopinath (2007), that the law of motion for \( \gamma_t \) is given by the following expression:

\begin{equation}
\log(\gamma_t) = \rho_\gamma \log(\gamma_{t-1}) + (1 - \rho_\gamma) \log(\gamma_{ss}) + \varepsilon_{\gamma,t},
\end{equation}

where \((\gamma_{ss} - 1)\) denotes the deterministic growth rate of the Total Factor Productivity in the Balanced Growth Path (BGP), \(|\rho_\gamma| < 1\) and \( \varepsilon_{\gamma,t} \) is a zero mean, i.i.d. process, drawn from a Normal distribution, with \( \text{Var}(\varepsilon_{\gamma,t}) = \sigma_\gamma^2 \).

Firms borrow from international lenders to pay a fraction \( \theta \) of their labor cost in advance of sales. Working capital is a within-period loan contracted at the beginning of each period and, as market for the services of capital is frictionless, firms can make the payments to the owners of capital, foreign lenders, at the end of the period when production is realized. Perfect competition eliminates profits in equilibrium so when the firm sells its output, \( y_t \), at the end of the period, the firm pays the end-of-period labor payments \((1 - \theta) w_t l_t\), that is, the remaining fraction \((1 - \theta)\) of its labor cost at the end of period \( t \); pays the rental services to the owners of capital \( r_t k_{t-1} \) and the repay the working capital loan (principal/plus interest \( R_t \theta w_t l_t \)).

\footnote{We will also solve for the case in which \( \theta = 0 \) to compare the difference in results with and without the}
Thus, the firm’s problem is to choose labor and capital and to maximize profits while taking output and input prices as well as interest rates on foreign debt as given and subject to working capital constraint:

\[ \max \pi_t = y_t - (1 - \theta)w_t l_t - r_t k_{t-1} - R_{t-1} \theta w_t l_t, \]

or equivalently,

\[ \max \pi_t = y_t - w_t l_t - r_t k_{t-1} - (R_{t-1} - 1) \theta w_t l_t. \]

The necessary and sufficient first-order conditions for an (interior) optimum are then given by

\[ z_t \alpha k_{t-1}^{\alpha-1} \Gamma_l^{1-\alpha} = r_t, \quad (21) \]

\[ z_t (1 - \alpha) k_{t-1}^{\alpha} \Gamma_l^{1-\alpha} l_t^{-\alpha} = w_t [1 + (R_{t-1} - 1) \theta], \quad (22) \]

where (21) and (22) show that marginal productivity of capital and labor equal their marginal cost, interest rate and the labor cost, respectively.

### 3.1.3 International financial markets

Agents in this economy can buy and sell international foreign assets in international financial markets. The evolution of the level of net holding of foreign assets is given by:

\[ b_t - \theta w_t l_t = T B_t + R_{t-1} [b_{t-1} - \theta w_t l_t], \quad (23) \]

where \( T B_t \) is trade balance at period \( t \), which in our model, is calculated as:

\[ T B_t = y_t - c_t - i_t - \varkappa(b_t). \quad (24) \]

The goods produced in-country that are not used in consumption, investment or bonds holding costs are the country’s net exports. The country’s net foreign asset position in period \( t \) is the household’s asset position, \( b_{t-1} \), net of the firm’s working capital debt, \( \theta w_t l_t \).

Following Neumeyer and Perri (2005) we decompose the real interest rate of Hong Kong into two components: an international interest rate and a country spread. The country risk,
\( D_t \), is computed as the ratio between Hong Kong real interest rates, \( R_t \), and international interest rates, \( R^*_t \):

\[
R_t = R^*_t D_t.
\tag{25}
\]

Taking logarithms on the above expression, we have:

\[
\log R_t = \log R^*_t + \log D_t.
\tag{26}
\]

The Hong Kong economy is a small open economy so that it cannot affect the world real interest rate \( R^*_t \). Our interest is in the effect of fluctuations in real interest rates on the economy, not in the source of those fluctuations.

The difference between the real interest rates observed in Hong Kong and the international interest rate is the country spread. As we saw in Section 2, we have used the real 3-month interbank offered rate that is provided by the HKMA dataset for the Hong Kong real interest rates. The international interest rate is the real US 3-month Treasury Bill rate. The real international interest rate has been calculated following the same procedure as for the Hong Kong interest rate. The country spread will capture the Hong Kong’s idiosyncratic default risk. Consequently, this assumption creates two sources of volatility in \( R_t \); one that is due to changes in the international preference of investors regarding risky assets, \( R^*_t \), and a second one that is the country spread, \( D_t \).

Regarding the stochastic process for each component of the interest rates, even though the correlation between the world interest rate and the country risk process is around 0.28, we assume, as in Neumeyer and Perri (2005), that both the international interest rate process and the country risk process are uncorrelated. In particular, we assume that the percentage deviation from trend of international interest rates follows an AR(1) process,

\[
\hat{R}^*_t + 1 = \rho \hat{R}^*_t + \varepsilon_{R^*,t+1},
\tag{27}
\]

where \( \hat{R}^*_t + 1 \) denotes the percentage deviation of the world interest rate from its trend, \( |\rho| < 1 \) and \( \varepsilon_{R^*,t+1} \) is a zero mean, i.i.d. process, drawn from a Normal distribution, with \( Var(\varepsilon_{R^*,t+1}) = \sigma^2_{R^*} \).

With regard to the country spread, we assume that it is driven only by exogenous factors (completely independent on the domestic factors\(^{17}\)). In particular, we assume that the

\(^{17}\)Several authors argue that the Asian financial crisis was due to external factors: i) Some argue that it was caused by a change in expectations from foreign investors (Obstfeld (1996), Radelet and Sachs (1998),
percentage deviation of trend from the country risk, \( \hat{D}_t \), follows an autoregressive process of the form

\[
\hat{D}_{t+1} = \rho_D \hat{D}_t + \varepsilon_{D,t+1}, \tag{28}
\]

where \( \hat{D}_{t+1} \) denotes the percentage deviation of the Hong Kong risk spread from its trend, \(|\rho_D| < 1\) and \( \varepsilon_{D,t+1} \) is a zero mean, i.i.d. process, drawn from a Normal distribution, with \( \text{Var}(\varepsilon_{D,t}) = \sigma_D^2 \).

3.1.4 The competitive equilibrium

Given initial conditions for the endogenous state variables \((k_{-1}, b_{-1})\) and exogenous state variables \((z_{-1}, \Gamma_{-1}, \gamma_{-1}, R^*_{-1}, D_{-1})\), an equilibrium is a sequence of quantities \(\{c_t, l_t, k_t, b_t, y_t, z_t, \gamma_t\}_{t=0}^{\infty}\) and prices \(\{w_t, r_t\}_{t=0}^{\infty}\), such that all economic agents maximize the objective functions subject to their constraints, taken as given the equilibrium prices, and all markets clear:

1) Given the equilibrium prices \(\{w_t, r_t\}_{t=0}^{\infty}\) and the exogenous sequence for \(\{R_t\}_{t=0}^{\infty}\), households’ choice \(\{c_t, l_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}\) maximize their welfare given their budget constraints.

2) Given the equilibrium prices \(\{w_t, r_t\}_{t=0}^{\infty}\) and the exogenous sequence for \(\{R_t\}_{t=0}^{\infty}\), firms’ choice \(\{k_t, l_t\}_{t=0}^{\infty}\) maximize their profits.

3) The law of motion for the exogenous state variables is given by the above processes (18), (19), (20), (27) and (28).

4) All markets clear.

The solution to the above competitive equilibrium is a non-linear system. A strategy to solve DSGE models consists of linearizing the first-order conditions and constraints around the deterministic balanced growth path. In order to guarantee that all variables are stationary in the balanced growth path equilibrium, some variables have to first be scaled by the cumulative growth rate of labor-augmenting technological progress \(\Gamma_{t-1}\). Once the variables have been normalized, the above FOC’s can be written as follows:

\[
\psi v(t^{(v-1)}) = \hat{w}_t, \tag{29}
\]

\[(\hat{c}_t - \psi t_v)^{-\sigma} \left[ 1 + \varphi \left( \frac{k_{t+1}}{k_{t-1}} - \gamma_{ss} \right) \right] = (30) \]

\[\beta \gamma^{-\sigma} E_t(\hat{c}_{t+1} - \psi t_v)^{-\sigma} \left[ 1 - \delta + r_{t+1} + \frac{\varphi}{2} \left( \frac{\gamma_{t+1} k_{t+1}}{k_t} - \gamma_{ss} \right)^2 \right], \]

\[(\hat{c}_t - \psi t_v)^{-\sigma} \left[ 1 + \chi \left( \frac{\hat{b}_t}{\hat{y}_t} - \bar{b} \right) \right] = \beta R_t \gamma^{-\sigma} E_t(\hat{c}_{t+1} - \psi t_{v+1})^{-\sigma}, \quad (31)\]

\[\hat{c}_t + \gamma_{t} \hat{k}_t - (1 - \delta) \hat{k}_{t-1} + \frac{\varphi}{2} \gamma_{t} \left( \frac{\gamma_{t} k_t}{k_{t-1}} - \gamma_{ss} \right)^2 + \gamma_{t} \hat{b}_t + \frac{\chi}{2} \hat{y}_t \left( \frac{\hat{b}_t}{\hat{y}_t} - \bar{b} \right)^2 = (32)\]

\[\hat{w}_t l_t + r_l \hat{k}_{t-1} + R_{t-1} \hat{b}_{t-1}. \]

From firms:

\[\hat{y}_t = z_t \hat{k}_{t-1} \left[ \gamma_{t} l_t \right]^{1-\alpha}, \quad (33)\]

\[z_t \alpha \hat{k}_{t-1}^{\alpha}(\gamma_{t} l_t)^{1-\alpha} = r_t, \quad (34)\]

\[z_t (1 - \alpha) \hat{k}_{t-1}^{\alpha}\gamma_t^{\alpha-1} l_t^{-\alpha} = \hat{w}_t [1 + (R_{t-1} - 1) \theta], \quad (35)\]

where: \(\hat{c}_t \equiv c_t / \Gamma_{t-1}, \hat{k}_{t-1} \equiv k_{t-1} / \Gamma_{t-1}, \hat{b}_{t-1} \equiv b_{t-1} / \Gamma_{t-1}, \hat{w}_t \equiv w_t / \Gamma_{t-1}, \) and \(\hat{y}_t \equiv y_t / \Gamma_{t-1}.\)

Appendix B shows the log-linearized first order conditions around the BGP equilibrium. We use the Schul decomposition to compute linear decision rules\(^{18}\). The balanced growth path equilibrium characterizes the long-run features of the economy in which the variables output, \(y_t\), consumption, \(c_t\), stock of physical capital, \(k_t\), stock of foreign bonds, \(b_t\), and the wage rate, \(w_t\), grow at an exogenous rate, \(\gamma_{ss}\), and the variables foreign interest rate, \(R_t\), physical capital return, \(r_t\), and labor, \(l_t\), remain constant. Once the policy functions are obtained, the aggregate variables are recovered, with the same length as is the data, and they are filtered (with the BP filter).

\(^{18}\)We base on Sims’ (2001) model that was further developed by Klein (2000). The MATLAB code for computing the equilibria is based on the one developed by Oviedo (2005a) and Schmitt-Grohé and Uribe (2004). The MATLAB code for computing solutions in this paper is available from the authors upon request.
4 Model solution

This section sets out the parameters used in the model and analyzes the quantitative implications of the model. Firstly, we obtain the conditions to be satisfied along the balanced growth path equilibrium. Secondly, we describe the calibration procedure for some of the parameter values. Thirdly, we describe the processes for the technology and interest rates. Finally, we report the moments generated by the model and compare them with the data.

4.1 Balanced Growth Path

As mentioned above, output, $y_t$, consumption, $c_t$, the stock of physical capital, $k_t$, the stock of foreign bonds, $b_t$, and the wage rate, $w_t$, grow at an exogenous rate on the balanced growth path equilibrium, $\gamma_{ss}$, and the foreign interest rate, $R_t$, physical capital return, $r_t$, and labor, $l_t$, remain constant.

On the balanced growth path equilibrium, the following conditions hold:

\[
\psi v l_s^{v-1} = \hat{w}_{ss},
\]

\[
1 = \beta \gamma_{ss}^{-\sigma}(1 - \delta + r),
\]

\[
1 = \beta R_{ss} \gamma_{ss}^{-\sigma},
\]

\[
\hat{c}_{ss} + [\gamma_{ss} - (1 - \delta)] \hat{k}_{ss} + \gamma_{ss} \hat{b}_{ss} = \hat{w}_{ss} l_{ss} + r \hat{k}_{ss} + R_{ss} \hat{b}_{ss},
\]

\[
\hat{y}_{ss} = \hat{k}_{ss}^{\alpha} [\gamma_{ss} l_{ss}]^{1-\alpha},
\]

\[
\alpha \hat{k}_{ss}^{-1} (\gamma_{ss} l_{ss})^{1-\alpha} = r_{ss},
\]

\[
(1 - \alpha) \hat{k}_{ss}^{\alpha} \gamma_{ss}^{1-\alpha} l_{ss}^{-\alpha} = \hat{w}_{ss} [1 + (R_{ss} - 1) \theta],
\]

from which we obtain the balanced growth path values for $\hat{w}_{ss}, l_{ss}, \hat{k}_{ss}, r_{ss}, \hat{c}_{ss}, \hat{y}_{ss}$ taking into account that $z_{ss} = 1^{19}$.

---

19 As it is standard in small open economies, the balanced growth path for $\hat{b}_{ss}$ is not uniquely determined. This is because the condition (38) only gives the relationship between the parameters $\beta$ and $\gamma_{ss}$, and the deterministic value for the Hong Kong interest rate $R_{ss}$, which is determined outside the model.
4.1.1 Parameter values

Some numbers need to be given to the parameters of the model in order to obtain numerical solutions. Some of the model parameters will be calibrated and some others will be set ad-hoc and not calibrated. The calibration exercise assigns values to the model’s parameters so that the deterministic balanced growth path equilibrium matches key averages from quarterly Hong Kong data. We will use data for Hong Kong for the 1982Q1-2004Q4 period. We assume that a time period in our model corresponds to one quarter. Parameter values are presented in Table 5.

First of all, the following condition on the parameters has to be satisfied $\beta \gamma_{ss}^{1-\sigma} < 1$ in order to guarantee that the utility function is well defined. This is the same condition that is obtained from the Transversality Condition (12).

The parameters that have been calibrated to mimic the balanced growth path equilibrium of the Hong Kong economy are: the exogenous growth rate $\gamma_{ss}$, the labor weight $\psi$ and the depreciation rate $\delta$. And the parameter calibrated to match some volatilities is the adjustment cost to capital $\varphi$. In particular, the parameter $\gamma_{ss}$ has been chosen such that it matches the average growth rate of the real output in Hong Kong between 1982Q1 and 2004Q4. The observed average growth rate of the real GDP in Hong Kong in the 1982Q1-2004Q4 period is 1.3%. Therefore, $\gamma_{ss} = 1.013$.

We assume a value for the discount factor $\beta = 0.987$, not too far from the more standard value in Real Business Cycles (Aguiar and Gopinath (2007a)), $\beta = 0.98$, and consistent with a not too high depreciation rate for the stock of capital. We set the utility curvature $\sigma = 2$ following Aguiar and Gopinath (2007a).

The calibration of the parameter $\delta$, that represents the constant depreciation rate of physical capital, is chosen such that the ratio of investment over GDP in the balanced growth path equilibrium matches the data. In particular, we have that,

$$\frac{\hat{i}_{ss}}{\hat{y}_{ss}} = (\gamma_{ss} - 1 + \delta) \frac{\hat{k}_{ss}}{\hat{y}_{ss}} = 0.27.$$  

Likewise, we have that,

$$\frac{\hat{k}_{ss}}{\hat{y}_{ss}} = \frac{\alpha}{r_{ss}}.$$

and that, from conditions (37) and (38), we know that,
Table 5: Parameter values

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Sign</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preference parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>( \beta )</td>
<td>0.987</td>
</tr>
<tr>
<td>Utility curvature</td>
<td>( \sigma )</td>
<td>2</td>
</tr>
<tr>
<td>Labor curvature</td>
<td>( \nu )</td>
<td>1.6</td>
</tr>
<tr>
<td>Labor weight</td>
<td>( \psi )</td>
<td>2.3282 ( (\theta = 0) )</td>
</tr>
<tr>
<td>Labor weight</td>
<td>( \psi )</td>
<td>1.863 ( (\theta = 1) )</td>
</tr>
<tr>
<td><strong>Technology parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technological progress growth</td>
<td>( \gamma )</td>
<td>1.3</td>
</tr>
<tr>
<td>a) No Working Capital</td>
<td>( \theta )</td>
<td>0.0</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>( \delta )</td>
<td>4.2</td>
</tr>
<tr>
<td>Capital exponent (production)</td>
<td>( \alpha )</td>
<td>0.4</td>
</tr>
<tr>
<td>Adjust. cost (capital)</td>
<td>( \varphi )</td>
<td>9.12</td>
</tr>
<tr>
<td>Adjust. cost (bonds)</td>
<td>( \chi )</td>
<td>( 10^{-3} )</td>
</tr>
<tr>
<td>b) Working Capital</td>
<td>( \theta )</td>
<td>1.0</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>( \delta )</td>
<td>5.4</td>
</tr>
<tr>
<td>Capital exponent (production)</td>
<td>( \alpha )</td>
<td>0.376</td>
</tr>
<tr>
<td>Adjust. cost (capital)</td>
<td>( \varphi )</td>
<td>1.61</td>
</tr>
<tr>
<td>Adjust. cost (bonds)</td>
<td>( \chi )</td>
<td>( 10^{-3} )</td>
</tr>
</tbody>
</table>

Note: The capital adjustment cost varies in the computed Experiments.
\[ r_{ss} = R_{ss} - 1 + \delta. \]

On the other hand, we have that from the condition (38),

\[ 1 = \beta R_{ss} \gamma_{ss}^{-\sigma}, \]

therefore,

\[ R_{ss} = \frac{\gamma_{ss}^\sigma}{\beta}. \]

In consequence,

\[ (\gamma_{ss} - 1 + \delta) \frac{\alpha}{\frac{\gamma_{ss}^\sigma}{\beta} - 1 + \delta} = 0.27. \]

On the other hand, we have to take into account that the labor-share can be defined as:

\[
\text{Labor share} = \frac{1 - \alpha}{1 + \theta \left( \frac{\gamma_{ss}^\sigma}{\beta} - 1 \right)}. \]

If there is no working capital requirement, \( \theta = 0 \), and assuming that the labor income share in total income is 0.6, then the capital exponent is \( \alpha = 0.4 \) and the quarterly constant depreciation rate is \( \delta = 0.042 \). Nevertheless, if there is a working capital requirement where all labor is paid in advance, \( \theta = 1 \), then the capital exponent is \( \alpha = 0.376 \) (it is lower because part of the income is used to pay the interest for the foreign borrowing) and the quarterly constant depreciation rate \( \delta = 0.054 \).

The parameter \( \psi \) has been chosen such that the average hours worked, per working-age population, for the period analyzed (1982-2004) is 28.3% (\( l_{ss} = 0.283 \)). From the condition (36), we have that,

\[ \psi = \frac{\bar{w}_{ss}^{(1-\nu)} l_{ss}}{\nu}, \]

where, we set the parameter \( \nu \) to 1.6 following Neumeyer and Perri (2005).\(^{20}\)

\(^{20}\)It is set in between 1.5 and 1.7 by different authors: Neumeyer and Perri set as 1.6, Mendoza as 1.5 and Correia et al. as 1.7.
\[
\frac{\hat{k}}{l_{ss}} = \left(\frac{\alpha}{\gamma}\right)^{1/(1-\alpha)} \gamma_{ss},
\]

and from equation (42), we have that,

\[
\hat{w}_{ss} = \frac{(1 - \alpha) \left(\frac{\hat{k}}{l_{ss}}\right)^{\alpha} \gamma_{ss}^{1-\alpha}}{1 + \theta \left(\frac{\gamma_{ss}}{\beta} - 1\right)} = \frac{(1 - \alpha) \left(\frac{\alpha}{\gamma}\right)^{\alpha/(1-\alpha)} \gamma_{ss}}{1 + \theta \left(\frac{\gamma_{ss}}{\beta} - 1\right)}.
\]

Consequently, we obtain that,

\[
\frac{\psi}{l_{ss}^{(1-\nu)}} = \frac{(1 - \alpha) \left(\frac{\alpha}{\gamma}\right)^{\alpha/(1-\alpha)} \gamma_{ss}}{1 + \theta \left(\frac{\gamma_{ss}}{\beta} - 1\right)}.
\]

Substituting \(l_{ss} = 0.283\), and taking into account the rest of parameters previously obtained, we obtain a value for \(\psi\) equal to 2.3282 if \(\theta = 0\) and a value for \(\psi\) equal to 1.863 if \(\theta = 1\).

The parameter of the adjustment cost to capital \(\varphi\) is chosen to replicate the relative volatility of investment to GDP (= 3.4).

The value for \(\bar{b}\) from the bond holding cost function has been chosen such that the consumption-GDP ratio is close to the observed one\(^{21}\). The parameter of the adjustment cost to holding bonds \(\chi\) is the same as in Schmitt-Grohé and Uribe (2003), and guarantees that the foreign bonds do not explode \((\chi = 10^{-3})\).

### 4.1.2 Dynamics of Interest rates and Productivity

With regard to the law of motion of the shock processes for the interest rates (domestic and international), they have been taken as exogenous processes determined outside the model. And, with regard to the two components of the total factor productivity (the transitory and the permanent components) their persistence has been taken outside the model, and their respective volatility have been calibrated.

As mentioned above, we decompose the Hong Kong real interest rate into two components: the world real interest rate \(R^*_t\), and the Hong Kong risk spread \(D\). Following Neumeyer and Perri (2005), we have estimated equations (27) and (28) by OLS, obtaining

\(^{21}\)Our business cycles properties are robust to changes to \(\bar{b}\).
their respective persistence parameters: $\rho_{R^*} = 0.903405$ and $\rho_D = 0.897795$. The volatility of the world real interest rate is 0.21% and the volatility of the Hong Kong interest rate is 0.68%.

With respect to productivity, we have calibrated the parameters $\sigma_{\epsilon_z}$ and $\sigma_{\epsilon_\gamma}$. Regarding the standard deviation of the innovation to these two components of the TFP, we have calibrated $\sigma_{\epsilon_z}$ in order to replicate the standard deviation of the output and $\sigma_{\epsilon_\gamma}$ in order to replicate the volatility of the growth rate of output. We find that, in order to mimic these two moments, $\sigma_{\epsilon_\gamma}$ has to be much higher than $\sigma_{\epsilon_z}$, 1.56 times higher if $\theta = 0$ and 1.86 times higher if $\theta = 1$. As for the parameter $\rho_z$, a standard value in the literature, 0.95, has been chosen. Finally, we have used the smallest positive value for the parameter $\rho_\gamma$, in order to be as close as possible to the observed autocorrelation for the growth of unfiltered output. However, we have not been able to capture a zero autocorrelation for the growth of unfiltered output as is noted.

The processes and the value of the parameters are shown in Table 6.

Table 6: Interest rates and productivity

<table>
<thead>
<tr>
<th>Process</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{R}^<em><em>t = \rho</em>{R^</em>} \hat{R}^<em><em>{t-1} + \epsilon</em>{R^</em>,t}$</td>
<td>$\rho_{R^*}$</td>
<td>0.903405</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\epsilon_{R^*}}$</td>
<td>0.21%</td>
</tr>
<tr>
<td>$\hat{D}<em>t = \rho_D \hat{D}</em>{t-1} + \epsilon_{D,t}$</td>
<td>$\rho_D$</td>
<td>0.897795</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\epsilon_D}$</td>
<td>0.55%</td>
</tr>
<tr>
<td>$\hat{z}<em>t = \rho_z \hat{z}</em>{t-1} + \epsilon_{z,t}$</td>
<td>$\rho_z$</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\epsilon_z}$</td>
<td>$\sigma_y$</td>
</tr>
<tr>
<td>$\hat{\gamma}<em>t = \rho</em>\gamma \hat{\gamma}<em>{t-1} + \epsilon</em>{\gamma,t}$</td>
<td>$\rho_\gamma$</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\epsilon_\gamma}$</td>
<td>$\sigma_{\Delta y}$</td>
</tr>
</tbody>
</table>

5 Results: Model Performance

In this section, we try to match the unconditional moments from the data to the moments obtained with the model under two scenarios. The first one does not consider the presence of working capital ($\theta = 0$). The other one takes into account the working capital requirements. We assume that 100% of the labor cost is paid in advance ($\theta = 1$). In both scenarios we calibrate the parameters $\sigma_{\epsilon_z}$, $\sigma_{\epsilon_\gamma}$ and $\varphi$ in order to replicate the output volatility, the volatility of the output growth and the relative volatility of investment to output observed
in the data. Table 7 compares some unconditional moments from the data and those implied by the model with and without the presence of working capital.

As can be seen, the results in terms of the correlation of the interest rates with output, it is too low if the assumption of working capital is not considered, and quite close to the observed one when the working capital requirements are included. This is because by reducing the presence of working capital, the negative impact that the interest rates have on labor demand is reduced\textsuperscript{22}. In terms of the rest of the moments, both models show similar results. The model overestimates the relative volatility of consumption (12\% if $\theta = 0$ and 20\% if $\theta = 1$)\textsuperscript{23}, net exports are much less countercyclical than in the data ($-0.01$ if $\theta = 0$ and 0.04 if $\theta = 1$, compared to $-0.33$ in the data), and the correlation of investment and output is too low ($0.35$ if $\theta = 0$ and 0.44 if $\theta = 1$, compared to 0.87 in the data). All in all, because the model with the assumption of working capital fits the correlation of interest rates with output better ($-0.12$ if $\theta = 0$ and $-0.31$ if $\theta = 1$, compared to $-0.32$ in the data), we take this model as the benchmark model in order to compute several experiments.

Before computing the experiments, we compare the obtained results from our benchmark model, to results from Neumeyer and Perri (2005) and Aguiar and Gopinath (2007a). Neumeyer and Perri (2005) finds that in a small open economy model with working capital requirements, transitory productivity shocks, international interest rate shocks and induced country risk premium shocks, the model is able to explain the negative correlation between interest rates and output and the negative correlation between the ratio of net exports to GDP and GDP for the Argentinean economy. However, if instead of endogenous country spread shocks, exogenous country risk premium shocks are considered, Neumeyer and Perri (2005) are not able to explain neither the magnitude of the correlation between real interest rates and output, nor the size of the correlation between the ratio of net exports to GDP and GDP. They need to assume that part of the volatility of interest rates is driven endogenously by domestic fundamentals. Concerning Aguiar and Gopinath´s (2007a) small open economy model, there are not financial frictions, but transitory and permanent productivity shocks are considered because the model with these two shocks is able to explain the high volatility of consumption with respect to income, and the negative correlation between the ratio of trade balance to output and output for Mexican economy.

\textsuperscript{22}Unlike Neumeyer and Perri (2005), we do not perform the analysis under the hypothesis that the country spread depends on domestic shocks. We only consider the case in which it is assumed that the country spread is driven by exogenous factors, since this case captures quite well the correlation between the Hong Kong interest rate and its GDP.

\textsuperscript{23}The assumption of Cobb-Douglas preferences does not change significantly this result.
Given the above results, we take the model with the presence of working capital as the benchmark model, and compute three different experiments in order to assess the role of the productivity shocks and the interest rate shocks: i) First, we consider a model economy without permanent productivity shocks, ii) then, we consider a model economy without international interest rate shocks and iii) finally, we consider a model economy with no country risk premium shock.

5.1 Relevance of permanent versus transitory components of the Total Factor Productivity

In Table 8 we show the business cycle characteristics if the permanent shock to the TFP is not considered. As already mentioned, the permanent shock of the TFP has to be higher than the transitory component in order to mimic the observed volatilities of the GDP and of the growth of the GDP. If we calibrate the value for $\sigma_{\epsilon_z}$ in order to mimic the volatility of the GDP, and we do not consider the permanent component of the shock to the TFP, then the volatility of the growth of the GDP is only 50% of its observed value (2.24% instead of 4.76%). Therefore, as in emerging economies, in order to explain the high output growth volatility in the Hong Kong economy, we need to include a stochastic trend. Furthermore, if the stochastic trend is not included, the results are quite bad for the volatility of consumption growth and for the volatility of investment growth rate. And, as expected, we obtain a lower volatility of consumption relative to GDP. The results for the rest of the statistics are very similar.

According to Aguiar and Gopinath (2007b), the Solow residual processes, which differ in emerging economies with respect to developed economies, might be due to deeper financial frictions in the economy. Chari, Kehoe and McGrattan (2007) (CKM) show that many frictions (including financial frictions) can be represented in a reduced form as a Solow residual. Therefore, the shocks to trend may be due to financial frictions, credit restrictions or any other market frictions. Working capital requirements and the existence of high capital adjustment costs are a form of financial frictions in an economy. Otsu (2010), in an analysis following CKM, finds that the investment wedge (a proxy used as financial friction) is the main friction in explaining the Sudden Stop episode in Hong Kong after the Asian financial crisis in 1997-98. The distortions in the foreign debt market are unimportant in the case of the Hong Kong economy. Nevertheless, the inclusion of financial frictions with the form of working capital requirements and high capital adjustment costs are not enough to explain the high volatility of output growth.
5.2 Relevance of the interest rate shocks

In this subsection, we analyze the business cycle characteristics when only the shock to the international interest rate, \( R^* \), is considered, and when only the shock to country risk premium is considered. Table 9 shows the results under these two scenarios. As expected, the correlation between the interest rate and the output is more negative when only the shock to country risk premium is considered. We obtain a correlation between the interest rate and output of \(-0.29\) when no international interest rate shocks are included, and a correlation of \(-0.23\) when no country spread shocks are included, compared to a negative correlation of \(-0.32\) in the data. If no risk premium is considered, the volatility of the interest rate is practically equal to its observed volatility (0.63 compared to 0.68 in the data). If no international interest rate shock is considered, the volatility of the interest rate is only 0.37\% of its observed volatility.

Without international interest rate shocks, the correlation between the trade balance to output and output is higher than in the benchmark case, but the correlation between the trade balance to output ratio and the interest rate is closer to data if no country spread shock is included. The results for the rest of the statistics is very similar.

Therefore, as in emerging economies, the country spread is the component that best explains the volatility and countercyclical behavior of the real interest rate in Hong Kong.
Table 7: Results. Data and Model with and without Working capital

<table>
<thead>
<tr>
<th></th>
<th>( \theta = 0 )</th>
<th>( \theta = 1 )</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a) Standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>( \sigma_y )</td>
<td>2.89</td>
<td>2.89</td>
</tr>
<tr>
<td>Output growth</td>
<td>( \sigma_{\Delta y} )</td>
<td>4.76</td>
<td>4.76</td>
</tr>
<tr>
<td>Consumption</td>
<td>( \sigma_{c/y} )</td>
<td>0.94</td>
<td>1.00</td>
</tr>
<tr>
<td>Investment</td>
<td>( \sigma_{I/y} )</td>
<td>3.40</td>
<td>3.40</td>
</tr>
<tr>
<td>Interest rate</td>
<td>( \sigma_R )</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>( \sigma_{\Delta c} )</td>
<td>4.28</td>
<td>4.25</td>
</tr>
<tr>
<td>Investment growth</td>
<td>( \sigma_{\Delta i} )</td>
<td>9.36</td>
<td>9.40</td>
</tr>
<tr>
<td><strong>b) Cross-correlations with Output</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>( \rho_{c,y} )</td>
<td>0.87</td>
<td>0.86</td>
</tr>
<tr>
<td>Investment</td>
<td>( \rho_{i,y} )</td>
<td>0.35</td>
<td>0.44</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>( \rho_{tb/y,y} )</td>
<td>-0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>( \rho_{R,y} )</td>
<td>-0.13</td>
<td>-0.31</td>
</tr>
<tr>
<td><strong>c) Cross-correlations with Interest rates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>( \rho_{c,R} )</td>
<td>-0.43</td>
<td>-0.61</td>
</tr>
<tr>
<td>Investment</td>
<td>( \rho_{i,R} )</td>
<td>-0.81</td>
<td>-0.82</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>( \rho_{tb/y,R} )</td>
<td>0.36</td>
<td>0.58</td>
</tr>
<tr>
<td><strong>d) Cross-correlations with Output growth</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption growth</td>
<td>( \rho_{\Delta c,\Delta y} )</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>Investment growth</td>
<td>( \rho_{\Delta i,\Delta y} )</td>
<td>0.45</td>
<td>0.42</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>( \rho_{tb/y,\Delta y} )</td>
<td>-0.02</td>
<td>-0.09</td>
</tr>
<tr>
<td><strong>e) Cross-correlations with Trade Balance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption growth</td>
<td>( \rho_{\Delta c,\tau y} )</td>
<td>-0.01</td>
<td>-0.12</td>
</tr>
<tr>
<td>Investment growth</td>
<td>( \rho_{\Delta i,\tau y} )</td>
<td>-0.01</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

Note: All moments are calculated from the model using parameters reported in Tables 5 and 6. The values for the parameters \( \sigma_{c_e}, \sigma_{c_z} \) and \( \varphi \) are equal to 1.11645\%, 1.74045\% and 7.2799, respectively, if \( \theta = 0 \), and are equal to 0.922\%, 1.7176725\% and 5.68687532, respectively, if \( \theta = 1 \). To compute the correlation of \( \tau y \) with output growth, or with consumption growth or with investment growth, since \( \tau y \) is BP filtered, and eliminated the first and last \( K \) values, the same is done with the unfiltered output growth, consumption growth and investment growth.
Table 8: Results. No Permanent Shock

<table>
<thead>
<tr>
<th></th>
<th>No Permanent shock</th>
<th>Benchmark Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a) Standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>$\sigma_y$</td>
<td>2.89</td>
<td>2.89</td>
</tr>
<tr>
<td>Output growth</td>
<td>$\sigma_{\Delta y}$</td>
<td>2.23</td>
<td>4.76</td>
</tr>
<tr>
<td>Consumption</td>
<td>$\sigma_{c/y}$</td>
<td>0.95</td>
<td>1.00</td>
</tr>
<tr>
<td>Investment</td>
<td>$\sigma_{I/y}$</td>
<td>3.40</td>
<td>3.40</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$\sigma_R$</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>$\sigma_{\Delta c}$</td>
<td>2.08</td>
<td>4.25</td>
</tr>
<tr>
<td>Investment growth</td>
<td>$\sigma_{\Delta I}$</td>
<td>8.83</td>
<td>9.40</td>
</tr>
<tr>
<td><strong>b) Cross-correlations with Output</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>$\rho_{c,y}$</td>
<td>0.85</td>
<td>0.86</td>
</tr>
<tr>
<td>Investment</td>
<td>$\rho_{i,y}$</td>
<td>0.46</td>
<td>0.44</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>$\rho_{tb/y,y}$</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>$\rho_{R,y}$</td>
<td>-0.29</td>
<td>-0.31</td>
</tr>
<tr>
<td><strong>c) Cross-correlations with Interest rates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>$\rho_{c,R}$</td>
<td>-0.64</td>
<td>-0.61</td>
</tr>
<tr>
<td>Investment</td>
<td>$\rho_{i,R}$</td>
<td>-0.81</td>
<td>-0.82</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>$\rho_{tb/y,R}$</td>
<td>0.56</td>
<td>0.58</td>
</tr>
<tr>
<td><strong>d) Cross-correlations with Output growth</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption growth</td>
<td>$\rho_{\Delta c,\Delta y}$</td>
<td>0.84</td>
<td>0.96</td>
</tr>
<tr>
<td>Investment growth</td>
<td>$\rho_{\Delta I,\Delta y}$</td>
<td>0.28</td>
<td>0.42</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>$\rho_{tb/y,\Delta y}$</td>
<td>-0.22</td>
<td>-0.09</td>
</tr>
<tr>
<td><strong>e) Cross-correlations with Trade Balance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption growth</td>
<td>$\rho_{\Delta c,tb/y}$</td>
<td>-0.29</td>
<td>-0.12</td>
</tr>
<tr>
<td>Investment growth</td>
<td>$\rho_{\Delta I,tb/y}$</td>
<td>-0.23</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

Note: All moments are calculated from the model using parameters reported in Tables 5 and 6. The values for the parameters $\sigma_z$, $\sigma_s$ and $\varphi$ are equal to 0.922%, 1.7176725% and 5.68687532, respectively, if there is permanent shock, and are equal to 1.27147255%, 0.0% and 5.77846672, respectively, if there is not permanent shock. To compute the correlation of $tb/y$ with output growth, or with consumption growth or with investment growth, since $tb/y$ is BP filtered, and eliminated the first and last $K$ values, the same is done with the unfiltered output growth, consumption growth and investment growth.
Table 9: Results: No Country Spread Shock, no International Interest rate Shock

<table>
<thead>
<tr>
<th></th>
<th>No Country Spread Shock</th>
<th>No International Interest rate Shock</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a) Standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>$\sigma_y$</td>
<td>2.89</td>
<td>2.89</td>
</tr>
<tr>
<td>Output growth</td>
<td>$\sigma_{\Delta y}$</td>
<td>4.76</td>
<td>4.76</td>
</tr>
<tr>
<td>Consumption</td>
<td>$\sigma_c / \sigma_y$</td>
<td>0.87</td>
<td>1.01</td>
</tr>
<tr>
<td>Investment</td>
<td>$\sigma_{\Delta t}/ \sigma_y$</td>
<td>3.40</td>
<td>3.40</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$\sigma_R$</td>
<td>0.25</td>
<td>0.63</td>
</tr>
<tr>
<td><strong>b) Cross-correlations with Output</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>$\rho_{c,y}$</td>
<td>0.96</td>
<td>0.86</td>
</tr>
<tr>
<td>Investment</td>
<td>$\rho_{i,y}$</td>
<td>0.58</td>
<td>0.44</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>$\rho_{tb/y,y}$</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>$\rho_{R,y}$</td>
<td>-0.23</td>
<td>-0.29</td>
</tr>
<tr>
<td><strong>c) Cross-correlations with Interest rates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>$\rho_{c,R}$</td>
<td>-0.35</td>
<td>-0.61</td>
</tr>
<tr>
<td>Investment</td>
<td>$\rho_{i,R}$</td>
<td>-0.62</td>
<td>-0.83</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>$\rho_{tb/y,R}$</td>
<td>0.16</td>
<td>0.27</td>
</tr>
<tr>
<td><strong>d) Cross-correlations with Output growth</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption growth</td>
<td>$\rho_{\Delta c,\Delta y}$</td>
<td>0.99</td>
<td>0.96</td>
</tr>
<tr>
<td>Investment growth</td>
<td>$\rho_{\Delta i,\Delta y}$</td>
<td>0.61</td>
<td>0.42</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>$\rho_{tb/y,\Delta y}$</td>
<td>-0.06</td>
<td>-0.05</td>
</tr>
<tr>
<td><strong>e) Cross-correlations with Trade Balance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption growth</td>
<td>$\rho_{\Delta c,tb/y}$</td>
<td>-0.07</td>
<td>-0.09</td>
</tr>
<tr>
<td>Investment growth</td>
<td>$\rho_{\Delta i,tb/y}$</td>
<td>-0.18</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

Note: All moments are calculated from the model using parameters reported in Tables 5 and 6. The values for the parameters $\sigma_{\epsilon}$, $\sigma_{\epsilon}$, and $\varphi$ are equal to 0.93%, 1.75% and 2.04, respectively, if there is no country spread shock, and are equal to 0.926%, 1.7207% and 5.782, respectively, if there is not international interest rate shock. To compute the correlation of $tb/y$ with output growth, or with consumption growth or with investment growth, since $tb/y$ is BP filtered, and eliminated the first and last K values, the same is done with the unfiltered output growth, consumption growth and investment growth.
6 Concluding Remarks

This paper analyzes the business cycle properties of the Hong Kong economy during the 1982Q1-2004Q4 period, which includes the financial crisis period suffered in Hong Kong in 1997-98 period. We show that the volatility of the output, of the growth rate of output and of real interest rates in Hong Kong are higher than their respective average volatilities among developed economies. These characteristics are obtained even when the financial crisis and post-financial crisis period is excluded from the analysis.

We build a stochastic neoclassical small open economy model that seeks to replicate the main business cycle characteristics of Hong Kong, and through which we try to quantify the role played by exogenous TFP (transitory and permanent) shocks and two components of the real interest rates (international and country risk spread shocks) in the business cycles characteristics of the Hong Kong economy.

We work on a description of the economy based on Neumeyer and Perri (2005) model and on Aguiar and Gopinath (2007a) model. Neumeyer and Perri (2005) describes a small open economy with working capital requirements, and (transitory) productivity and interest rate (international and country risk premium) shocks. They are able to explain the negative correlation between interest rates and output observed in the Argentinean economy (which is much more countercyclical than in Hong Kong). Aguiar and Gopinath (2007a) build a small open economy model without working capital requirements, and with transitory and permanent productivity shocks, and they are able to explain consumption volatility exceeding income volatility, and strongly countercyclical current accounts in the Mexican economy. In a more recent paper, Chang and Fernández (2010) develop a small open economy model with financial frictions (as in Neumeyer and Perri (2005)) and transitory and permanent shocks to TFP (as in Aguiar and Gopinath (2007a)). They conclude that financial frictions (working capital requirements and spread linked to expected future productivity) seem to be more important in explaining the Mexican data.

We find, firstly, that, in order to replicate the high volatility of the growth of the Hong Kong GDP, the volatility of the trend has to be higher than the volatility of the transitory fluctuations around the trend. Furthermore, the model with permanents shocks to TFP better captures some other second moments (volatility of consumption growth and volatility of investment growth), without making worse in the rest of the statistics. Secondly, that the

\[24\text{In Hong Kong the volatility of consumption with respect to the volatility of income is very close to the observed among developed economies. However, the volatility of the growth of the output is much higher than observed among developed economies.} \]
country risk premium is the component that explains the high volatility of the Hong Kong interest rates. And, hence, is the responsible for the fact that interest rates in Hong Kong are countercyclical. Thirdly, that both working capital requirements and permanent shocks to TFP might be behind of some of the characteristics of the business cycles in Hong Kong.
7 Appendix A: Data description

The data we use to compute the business cycle characteristics of the Hong Kong economy was obtained from the Census and Statistics Department from the Hong Kong Government.\(^{25}\) We analyze the business cycle components of several series. The variables belong to two main categories: GDP components and interest rates. We study real variables (deflated by GDP deflator) and all series, except net exports and interest rates, have been transformed into logarithms before being filtered using the Band Pass filter to extract the cyclical components of the variables. The analysis has been done using annual and quarterly data.\(^{26}\) In this paper we only show results from quarterly data analysis.

Data for emerging and developed economies is taken from Aguiar and Gopinath (2004).

**National Accounts** The period covered goes from 1982Q1 to 2004Q4 for quarterly data. We choose this period to be able to compare with data from Aguiar and Gopinath (2004).

Data for private consumption, Government consumption, investment, exports, imports and GDP is obtained from quarterly Hong Kong Census and Statistics dataset. Investment is gross fixed capital formation plus change in inventories. Net exports is constructed as the difference between exports and imports.

For comparison with other countries, we also use data from Aguiar and Gopinath (2004).

**Real interest rates** For the analysis of the cyclical behavior of interest rates, three interest rates as representative of the short-term money market rate have been chosen:

1) Three-month real interest rate (Hong Kong dollar Inter-bank Offered rate at three months. It refers to the middle closing rate quoted by the Standard Chartered Bank in the interbank money market) from the Hong Kong Monetary Authority (HKMA). The periods covered for the analysis of the real interest rate series go from 1982Q1 to 2004Q4.

2) Best Lending interest rate (rate quoted by the Hong Kong and Shanghai Banking Corporation Limited). This interest rate is used by Leung and Suen in their analysis of the business cycles in Hong Kong. The periods covered for the analysis of the real interest rate series go from 1982Q1 to 2004Q4. We do not show the results with this interest rate in the paper. Available upon request.

\(^{25}\)Further information in the website of the Census and Statistics Department of Hong Kong at http://www.censtatd.gov.hk
\(^{26}\)For annual data, the HP filter with \(\lambda = 400\) and \(\lambda = 100\) and BP with \(K = 3\) have been used to detrend the series. For quarterly data, the HP filter with \(\lambda = 1600\) and the BP filter with \(K = 8\) have been used.
3) Treasury Bill Rate from IFS data. The periods covered for the analysis of the real interest rate series go from 1993Q4-2007Q3.

The real interest rate is obtained by dividing nominal interest rate by the expected inflation rate:

\[ 1 + r_{t+1} = \frac{1 + i_{t+1}}{1 + \pi_{t+1}}, \]

where \( r_{t+1} \) is real interest rate, \( i_{t+1} \) is nominal interest rate and \( \pi_{t+1} \) is inflation rate. Expected inflation in period \( t \) is calculated as the average of previous three quarters’ inflation rate of the GDP deflator.

8 Appendix B: Log-linearization of the model

A strategy to solve DSGE models involves the linearization of first-order conditions and constraints by means of a first-order Taylor series expansion around the deterministic balanced growth path (BGP). We denote by \( \tilde{x}_t = \log \left( \frac{x_t}{\bar{x}} \right) \) the log-deviation of \( x_t \) from its steady state value \( \bar{x} \) and we take first-order log-linearization of the model\(^27\). The linearized equations of the model are shown below.

1.- Labor. We log-linearize equation (36) and (42) around the BGP and obtain:

\[ z_t (1 - \alpha) \frac{k_t^{\alpha} (1-\alpha) l_t^{(1-\alpha)}}{[1 + (R_{t-1} - 1)\theta]} = \tilde{w}_t = -\frac{U_{l+1}}{U_{l+1}} = \psi v l_t^{(v-1)} [1 + (R_{t-1} - 1)\theta], \]

\[ z_t (1 - \alpha) \frac{\hat{k}_t (1-\alpha) l_t^{(1-\alpha)}}{[1 + (R_{t-1} - 1)\theta]} = \psi v l_t^{(v-1)} [1 + (R_{t-1} - 1)\theta], \]

\[ (1 - \alpha) \left[ \hat{k}_t^{\alpha} (1-\alpha) l_t^{1-\alpha} dz_t + z_t \alpha \hat{k}_t^{\alpha-1} l_t^{1-\alpha} \gamma_t^{(1-\alpha)} d\hat{k}_t + \ldots \right] = \psi v (v-1) l_t^{(v-2)} [1 + (R_{t-1} - 1)\theta] d\hat{l}_t + \psi v l_t^{(v-1)} \theta dR_{t-1}. \]

Next, we multiply and divide by \( z_t, \hat{k}_t, l_t, \gamma_t \) and \( R_{t-1} \) where necessary and evaluate at

\(^27\)We have to remind that the following variables \( \hat{c}_t \equiv c_t / \Gamma_{t-1}, \hat{k}_{t-1} \equiv k_{t-1} / \Gamma_{t-1}, \hat{b}_{t-1} \equiv b_{t-1} / \Gamma_{t-1}, \hat{w}_t \equiv w_t / \Gamma_{t-1}, \) and \( \hat{y}_t \equiv y_t / \Gamma_{t-1} \) are measured in efficiency units. Whereas, the variables \( l_t, r_t, R_t \) and \( \gamma_t \), are not measured in efficiency units.
the BGP:

\[(1-\alpha) \left[ \hat{k}_{ss}^{\alpha} \gamma_{ss}^{(1-\alpha)} l_{ss}^{-\alpha} \hat{z}_t + \alpha \hat{k}_{ss}^{\alpha} \gamma_{ss}^{(1-\alpha)} \tilde{k}_t + \hat{\gamma}_{ss} (1-\alpha) l_{ss}^{-\alpha} \hat{\gamma}_t + (1-\alpha) \hat{k}_{ss}^{\alpha} \gamma_{ss}^{(1-\alpha)} l_{ss}^{-\alpha} \gamma_t \right] = \ldots \psi v(v-1) l_{ss}^{(v-1)} [1 + (Rss - 1) \theta] \tilde{l}_t + \psi vl_{ss}^{(v-1)} \theta Rss \tilde{R}_{t-1}.
\]

Rearranging terms,

\[
\tilde{l}_t = \left[ (v-1) \psi vl_{ss}^{(v-1)} [1 + (Rss - 1) \theta] + (1-\alpha) \alpha \hat{k}_{ss}^{\alpha} \gamma_{ss}^{(1-\alpha)} l_{ss}^{-\alpha} \right]^{(1-\alpha)} \left( \tilde{z}_t + \alpha \tilde{k}_t + (1-\alpha) \tilde{\gamma}_t \right),
\]

\[
\tilde{l}_t = \left[ (1-\alpha) \hat{k}_{ss}^{\alpha} \gamma_{ss}^{(1-\alpha)} l_{ss}^{-\alpha} \right] \left( \tilde{z}_t + \alpha \tilde{k}_t + (1-\alpha) \tilde{\gamma}_t \right) - \psi vl_{ss}^{(v-1)} \theta Rss \tilde{R}_{t-1}.
\]

As in the BGP \((1-\alpha) \hat{k}_{ss}^{\alpha} \gamma_{ss}^{(1-\alpha)} l_{ss}^{-\alpha} = \psi vl_{ss}^{(v-1)} [1 + (Rss - 1) \theta],
\]

\[
\tilde{l}_t = \frac{1}{(v-1) + \alpha} \left( \tilde{z}_t + \alpha \tilde{k}_t + (1-\alpha) \tilde{\gamma}_t - \tilde{R}_{t-1} \right).
\]

2.- Euler equation for capital:

\[
(\hat{c}_t - \psi l_{t}^{v})^{-\sigma} \left[ 1 + \varphi \frac{\gamma_k\tilde{k}_t}{\tilde{k}_{t-1}} - \gamma_{ss} \right] = \ldots
\]

\[
\beta \gamma_t^{-\sigma} E_t (\hat{c}_{t+1} - \psi l_{t+1}^{v})^{-\sigma} \left[ 1 - \delta + r_{t+1} + \varphi \left( \frac{\gamma_{t+1}\tilde{k}_{t+1}}{\tilde{k}_t} \right)^2 - \gamma_{ss}^2 \right] .
\]

We log-linearize the above equation around the BGP and obtain:
\[-\sigma (c_t - \psi t^v) -1 \left[ 1 + \varphi \left( \frac{\gamma t k_t}{k_{t-1}} - \gamma_{ss} \right) \right] (d c_t - \psi v t^{v-1} d t_t) + \ldots \]

\[(c_t - \psi t^v)^-\sigma \varphi \left( \frac{\hat{k}_t}{k_{t-1}} d \gamma_t + \frac{\gamma_t}{k_{t-1}} d \hat{k}_t - \frac{\gamma_t}{k_{t-1}} d \hat{k}_{t-1} \right) = \ldots \]

\[-\sigma \gamma_t^{- \sigma} \beta E_t (c_t+1 - \psi t^v)^{\sigma} \left[ 1 - \delta + r_{t+1} + \frac{\varphi}{2} \left( \left( \frac{\gamma_{t+1} \hat{k}_{t+1}}{k_t} \right)^2 - \gamma_{ss}^2 \right) \right] d \gamma_t - \ldots \]

\[\beta \gamma_t^{- \sigma} (c_t+1 - \psi t^v)^{-1} \left[ dr_{t+1} + \varphi \left( \frac{\gamma_t \hat{k}_{t+1}}{k_t} \right) \left( \frac{\hat{k}_{t+1}}{k_t} d \gamma_{t+1} + \frac{\gamma_t}{k_t} d \hat{k}_{t+1} - \frac{\gamma_t}{k_t^2} d \hat{k}_t \right) \right]. \]

Next, we multiply and divide by \(\hat{c}_t, \hat{c}_{t+1}, l_t, l_{t+1}, \hat{k}_{t-1}, \hat{k}_t, \hat{k}_{t+1}, \gamma_t, \gamma_{t+1}\) and \(r_{t+1}\) where necessary and evaluate at the BGP:

\[-\sigma (c_{ss} - \psi v l_{ss})^{-1} \left[ c_{ss} c_t - \psi v l_{ss}^{-1} l_t \right] + (c_{ss} - \psi v l_{ss})^{-\sigma} \varphi \gamma_{ss} \left( \gamma_t + \hat{k}_t - \hat{k}_{t-1} \right) = \ldots \]

\[-\sigma \gamma_{ss}^{- \sigma} \beta (c_{ss} - \psi v l_{ss})^{- \sigma} \left[ 1 - \delta + r_{ss} \right] \gamma_t - \ldots \]

\[\sigma \beta \gamma_{ss}^{- \sigma} (c_{ss} - \psi v l_{ss})^{- \sigma} \left[ 1 - \delta + r_{ss} \right] \left( \gamma_t c_{t+1} - \psi v l_{ss}^{-1} l_{t+1} \right) + \ldots \]

\[\beta \gamma_{ss}^{- \sigma} (c_{ss} - \psi v l_{ss})^{- \sigma} \left[ r_{ss} \gamma_{t+1} + \varphi \gamma_{ss}^2 \left( \gamma_{t+1} + \hat{k}_{t+1} - \hat{k}_t \right) \right]. \]

Rearranging terms, we obtain the following log-linear equation:

\[-\sigma (c_{ss} - \psi v l_{ss})^{- \sigma} \left[ c_{ss} c_t - \psi v l_{ss}^{-1} l_t \right] - (c_{ss} - \psi v l_{ss})^{-\sigma} \varphi \gamma_{ss} k_{t-1} + \ldots \]

\[(c_{ss} - \psi v l_{ss})^{-\sigma} \varphi \gamma_{ss} \left[ 1 + \beta \gamma_{ss}^{-1} \right] \hat{k}_t + (c_{ss} - \psi v l_{ss})^{-\sigma} \gamma_{ss} \left[ \varphi + \beta \sigma \gamma_{ss}^{- \sigma} (1 - \delta + r_{ss}) \right] \gamma_t = \ldots \]

\[-\sigma \beta \gamma_{ss}^{- \sigma} (c_{ss} - \psi v l_{ss})^{- \sigma} \left[ 1 - \delta + r_{ss} \right] \left( \gamma_t c_{t+1} - \psi v l_{ss}^{-1} l_{t+1} \right) + \beta \gamma_{ss}^{- \sigma} (c_{ss} - \psi v l_{ss})^{- \sigma} r_{ss} \gamma_{t+1} + \ldots \]

\[\beta \gamma_{ss}^{- \sigma} (c_{ss} - \psi v l_{ss})^{- \sigma} \varphi \gamma_{ss}^2 \left( \gamma_{t+1} + \hat{k}_{t+1} \right). \]

Simplifying, we have,
\[ -(\hat{c}_{ss} - \psi v l_{ss})^{-1} \left[ \hat{c}_{ss} \hat{c}_t - \psi v l_{ss} \hat{l}_t \right] - \sigma \hat{c}_t \hat{c}_{t+1} - \psi v l_{ss} \hat{l}_{t+1} + \ldots \]

\[ \varphi \gamma_{ss} \left[ 1 + \beta \gamma_{ss}^{-1} \right] \tilde{k}_t + [\varphi \gamma_{ss} + \sigma] \tilde{\gamma}_t = \ldots \]

\[ -(\hat{c}_{ss} - \psi v l_{ss})^{-1} \left[ \hat{c}_{ss} \hat{c}_{t+1} - \psi v l_{ss} \hat{l}_{t+1} \right] + \beta \gamma_{ss}^{-1} r_{ss} \tilde{r}_{t+1} + \ldots \]

Next, we multiply and divide by \( \hat{c}_t, \hat{c}_{t+1}, l_t, l_{t+1}, \hat{b}_t, \tilde{b}_t, \gamma_t \) and \( R_t \) and evaluate at the BGP:

\[ -(\hat{c}_{ss} - \psi v l_{ss})^{-1} \left[ \hat{c}_{ss} \hat{c}_t - \psi v l_{ss} \hat{l}_t \right] + (\hat{c}_{ss} - \psi v l_{ss})^{-1} \chi \left[ \hat{b}_t - \tilde{b}_t \right] = \ldots \]

\[ -(\hat{c}_{ss} - \psi v l_{ss})^{-1} \varphi \gamma_{ss}^{-1} [\hat{c}_{ss} \hat{c}_{t+1} - \psi v l_{ss} \hat{l}_{t+1}] + \beta R_{ss} \gamma_{ss}^{-1} (\hat{c}_{ss} - \psi v l_{ss})^{-1} \tilde{R}_t. \]

Simplifying, and taking into account that \( \beta R_{ss} \gamma_{ss}^{-1} = 1 \), we have:

\[ (\hat{c}_t - \psi v l_t)^{-\sigma} \left[ 1 + \chi \left( \frac{b_t}{\tilde{b}_t} - \tilde{b}_t \right) \right] = \beta R_t \gamma_t^{-\sigma} E_t (\hat{c}_{t+1} - \psi v l_{t+1})^{-\sigma}. \]
\(-\sigma(\hat{c}_{ss} - \psi v \ell^v)\) \(\hat{c}_c + \hat{c}_t - \psi v \ell^v \hat{t}_t + \chi \hat{b}_t - \hat{y}_t = \ldots\)

\(-\sigma \hat{\gamma}_t - \sigma(\hat{c}_{ss} - \psi v \ell^v)\) \(\hat{c}_c + \hat{c}_t - \psi v \ell^v \hat{t}_t + \chi \hat{b}_t - \hat{y}_t = \ldots\)

4.- Household’s budget constraint:

\(\hat{c}_t + \gamma_t \hat{k}_t - (1 - \delta) \hat{k}_{t-1} + \frac{\varphi}{2} \hat{k}_{t-1} \left( \frac{\gamma_t \hat{k}_t}{\hat{k}_{t-1}} - \gamma_{ss} \right)^2 + \ldots\)

\(\hat{\gamma}_t \hat{b}_t + \frac{\hat{x}}{\gamma_t} \left( \frac{\hat{b}_t}{\hat{y}_t} - \hat{b} \right)^2 = \hat{w}_t l_t + r_t \hat{k}_{t-1} + R_{t-1} \hat{b}_{t-1}.\)

We log-linearize the above equation around the BGP and obtain:

\(d\hat{c}_t + \hat{k}_t d\gamma_t + \gamma_t d\hat{k}_t - (1 - \delta)d\hat{k}_{t-1} + \frac{\varphi}{2} \left( \frac{\gamma_t \hat{k}_t}{\hat{k}_{t-1}} - \gamma_{ss} \right)^2 d\hat{k}_{t-1} + \ldots\)

\(\varphi \hat{k}_{t-1} \left( \frac{\gamma_t \hat{k}_t}{\hat{k}_{t-1}} - \gamma_{ss} \right) \left( \frac{\hat{k}_t}{\hat{k}_{t-1}} d\gamma_t + \frac{\gamma_t}{\hat{k}_{t-1}} d\hat{k}_t - \frac{\gamma_t \hat{k}_t}{\hat{k}_{t-1}} d\hat{k}_{t-1} \right) + \ldots\)

\(\hat{\gamma}_t d\hat{b}_t + \gamma_t d\hat{b}_t + \frac{\hat{x}}{2} \left( \frac{\hat{b}_t}{\hat{y}_t} - \hat{b} \right)^2 d\hat{y}_t + \chi \left( \frac{\hat{b}_t}{\hat{y}_t} - \hat{b} \right) \left( \frac{\hat{y}_t^- d\hat{b}_t - \hat{b}_t}{\hat{y}_t^- d\hat{y}_t} \right) = \ldots\)

\(l_t d\hat{\ell}_t + \hat{\ell}_t d\gamma_t + \hat{k}_{t-1} d\gamma_t + r_t d\hat{k}_{t-1} + \hat{b}_{t-1} dR_{t-1} + \hat{R}_{t-1} d\hat{b}_{t-1}.\)

Next, we multiply and divide by \(\hat{c}_t, l_t, \hat{k}_t, \hat{k}_{t-1}, \hat{b}_t, \hat{y}_t, \hat{w}_t, r_t, \gamma_t\) and \(R_{t-1}\) and evaluate at the BGP:

\(\hat{c}_{ss} \hat{c}_t + \hat{k}_{ss} \gamma_\hat{\gamma} \hat{\gamma}_t + \gamma_{ss} \hat{b}_{ss} \hat{k}_t - (1 - \delta) \hat{k}_{ss} \hat{k}_{t-1} + \hat{b}_{ss} \gamma_{ss} \hat{\gamma}_t + \gamma_{ss} \hat{b}_{ss} \hat{b}_t = \ldots\)

\(\hat{w}_{ss} l_{ss} \hat{w}_t + \hat{w}_{ss} l_{ss} \hat{t}_t + \hat{k}_{ss} r_{ss} \hat{r}_t + r_{ss} \hat{k}_{ss} \hat{k}_{t-1} + \hat{b}_{ss} R_{ss} \hat{R}_{t-1} + R_{ss} \hat{b}_{ss} \hat{b}_{t-1}.\)

Rearranging terms, we have
\[ \hat{c}_{ss} \hat{c}_t + \left( \hat{k}_{ss} + \hat{b}_{ss} \right) \gamma_{ss} \hat{\gamma}_t + \gamma_{ss} \left( \hat{k}_{ss} \hat{k}_t + \hat{b}_{ss} \hat{b}_t \right) - \left( r_{ss} + (1 - \delta) \right) \hat{k}_{ss} \hat{k}_{t-1} = \ldots \]

\[ \hat{w}_{ss} \hat{l}_{ss} \hat{w}_t + \hat{w}_{ss} \hat{l}_{ss} \hat{l}_t + \hat{k}_{ss} \hat{r}_{ss} \hat{r}_t + \hat{b}_{ss} \hat{R}_{ss} \hat{R}_{t-1} + \hat{R}_{ss} \hat{b}_{ss} \hat{b}_{t-1}. \]

9 Appendix C: Solving Dynamic General Equilibrium Models

We solve the problem using Linear Perturbation methods available from Schmitt-Grohé and Uribe at the webpage: www.columbia.edu/~mu2166/2nd_order.htm. Once the model has been solved, we have simulated the model 2000 times, and we have obtained all statistics as the average of the 2000 statistics. All model series of the same length as the data sample.
References


