A Probabilistic Voting Model of Progressive Taxation with Incentive Effects *

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Abstract

This paper shows conditions under which a marginally progressive income tax emerges as the outcome of political competition between two parties, when labor is elastically supplied and candidates are uncertain about voters' choice at election day. Assuming the elasticity of labor is decreasing on marginal wage; following Coughlin and Nitzan (1981) only marginal progressive taxes are played by both candidates in equilibrium. If, instead, we adopt Lindbeck and Weibull (1989) probabilistic voting model, the equilibrium tax schedule will be progressive as long as the political power of the rich voter is sufficiently small. The degree of progressivity decreases with population polarization.

*JEL classification: D3, D63, D72, H24.

Keywords: Political economy, progressive taxation, elastic labor supply.

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*I am very grateful to Enriqueta Aragonès, Georges Casamatta, Helmuth Cremer, Philippe De Donder and participants of the XXXIII Economic Analysis Symposium for helpful comments and suggestions. The usual disclaimer applies.

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1 Introduction

The question, "Why do progressive taxes emerge in industrialized countries?", dates from Mirrlees seminal paper (1971). He showed that marginal progressive tax schedules, as we have in industrialized societies, were hardly optimal, unless we had a small elasticity of labor supply. A growing literature on political economy of taxation inspired by Roberts (1977), Romer (1975) and Meltzer and Richard (1981), questioned whether high marginal taxes could be part of a political equilibrium. Though, disincentives effects from taxation were taken into account, restrictions on the tax schedule were imposed making difficult to study tax progressivity. Among those that study tax progressivity, few of them considered disincentive effects from taxation (De Donder and Hindriks, 2003) the main literature built under the exogenous income hypothesis, see for example Marhuenda and Ortuño-Ortín (1995), Roemer (1999), Carbonell and Klor (2003) and Carbonell and Ok (2007).

The starting literature on the political economy of taxation assumed a proportional income tax. Under some conditions on preferences such as single-crossing a Condorcet winner (CW) exists and both Downsian candidates play the CW tax rate in equilibrium. In the absence of labor disincentives (inelastic labor supply) if the median income is below the mean, then, the equilibrium marginal tax rate equals 100%. Even if endogenous labor supply is assumed taxes are still strictly positive and increasing with inequality, defined as the ratio of average income to median income.

Nevertheless, assuming a linear income tax schedule could not help us to understand the fact that most industrial economies marginal tax rates are increasing with income. The aim of this paper is to understand why there is a democratic demand for income tax progressivity. In order to have a tax schedule that allows for (marginal) progressivity we
need at least three parameters to vote for." One parameter specifies the lump-sum transfer (or level of public good), other, the linear tax rate, and the last capturing the concavity (regressivity) or convexity (progressivity) of the tax schedule. Thus, we are facing a multidimensional voting model. Conditions to have a CW in models with a multidimensional policy space are known to be very restrictive. For the quadratic tax function De Donder and Hindriks (2003) and Hindriks (2001) show that it is hard to avoid voting cycles. Other approaches than the direct democracy approach should be considered. Carbonell and Klor (2003) in a citizen candidate model found that, under some conditions, only marginal increasing tax rates are implemented in equilibrium. Roemer (1999) developed the PUNE concept (Party Unanimity Nash Equilibrium). The platform chosen by the party is the outcome of intraparty negotiation among party members. In equilibrium both parties announce marginal progressive taxes. In this paper we adopt the probabilistic voting model introduced by Coughlin and Nitzan (1981) and Coughlin (1992), and microfunded afterwards by Lindbeck and Weibull (1987). When voters cast their ballots in favor of one or another candidate they consider issues other than the economic issue, for instance ideology. Still, the higher the difference in economic utility, the higher will be the probability that a given voter favors the candidate that brings him the highest (economic) utility. Conditions for existence of equilibrium are less restrictive in the Coughlin model. A CW needs not to exist and, when it does exist, the equilibrium tax schedule does not need to coincide with the CW tax schedule.

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Note that if we restrict the policy space to tax functions ordered by Lorenz dominance, that is the case is after-tax income can be represented as

\[ x_i = (y_i)^{1-\tau} \left( \hat{y} \right)^\tau, \]

where \( \hat{y} \) is common to all agents (it is determined so that average post-tax income equals per-capita income), \( y_i \) is pre-tax income and \( \tau \) is the tax parameter. A single parameter is enough to describe whether the tax schedule is marginal-rate progressive \((0 \leq \tau \geq 1)\), or regressive \( \tau \leq 0 \). See for instance Bénabou (2000).
The probabilistic model can be understood as the outcome of a political process where voters choose probabilistically between candidates, with the probability to vote for one candidate increasing in the utility difference. The outcome of such a political process, as stressed by Coughlin and Nitzan (1981) in their Theorem 1, involves the maximization of a Nash welfare function. Indeed, in both models, Coughlin (CN) and Lindbeck and Weibull (LW), the equilibrium income tax maximizes some welfare function. In this sense the equilibrium income tax is efficient; it is on the economy’s Utility frontier. We show which conditions on the welfare function and the labor supply need to be satisfied for a marginal progressive tax to emerge in equilibrium. Moreover, if those conditions are met, then, the progressivity degree will decrease with population polarization.

For simplicity, we assume quasi-linear preferences, for which income effects are zero. Therefore, when the tax rate paid by a given group increases, labor supply unambiguously decreases. Labor supply responses will add another mechanism for which a given vote is easier to catch. If the elasticity of labor is decreasing in marginal wage, as assumed, then, we could tax heavily the rich relative to the poor since the former decreases less his labor supply in response of an increase in the tax. In this sense there is more scope for tax progressivity than in the fixed (exogenous) income model. It is worth noting that there are little estimates on the elasticity of labor supply by income groups. Saez (2004) finds that upper middle income families and individuals do not appear to be sensitive to taxation, which supports our assumption of lower elasticity for the middle and high income groups. Nevertheless, significant elasticities are found at the very top of the income distribution. Whether those externalities could be explained solely by the evolution of marginal tax rates is not clear, given the heterogeneity in size of responses overtime.

The probabilistic voting model would bring credible predictions in any of these three
scenarios: Voters vote probabilistically, candidates are uncertain about voters’ choices or we have interest groups representing voters that compete for influence.

The rest of the paper goes as follows: Section 2 presents the model. Section 3 describes the labor supply decision of voters given the implemented tax schedule. In section 4 we describe the preferences of the different income groups over tax schedules. In section 5 we describe the political competition stage and the main results of the paper. Section 6 concludes.

2 The Model

We develop a static model of political competition between two Downsian parties, $A$ and $B$. Candidates or parties are uncertain about how the economic preferences of voters translate into party preferences. Parties announce simultaneously a policy platform $t^C$, $C = A, B$, a vector of marginal income tax rates, that maximizes their probability of winning. They commit to the platform announced. The party holding the majority of votes wins the election. Once the equilibrium platform is implemented voters make labor decisions. We solve the model backwards.

2.1 The Probabilistic Voting Model

The probabilistic voting model developed by Coughlin relaxes one of the assumptions of the traditional Downsian model: Candidates are certain about what voters choices will be in response of their announced platforms.

In Coughlin and Nitzan (1981) and Coughlin (1992) even after voters have learned the decisions of both of the candidates in the race, candidates are uncertain about voters’ actions at the election day. This would also be the case if voters’ choices were stochastic in
Two possible interpretations of the Coughlin model are that voters do not vote deterministically but they are still rational: they vote with higher probability for the candidate whose policy platform brings them the highest utility.

This raises the question of why voters do not vote according to their economic preferences.

This leads to the second interpretation of the Coughlin model, where voters vote indeed deterministically but there are other issues apart from the economic issue, then, they may not vote for the party that promises them the best economic platform. Here voters are *ideological*.

Candidates use a logit model to infer voters' selection probabilities. In an economy with $J$ voters the probability that a voter of group $j$ votes for candidate $A$ equals the relative utility $j$ derives from $A$ platform with respect to the utility he derives from $B$’s platform,

$$
\pi^A_j (t^A, t^B) = \frac{U_j(t^A)}{U_j(t^A) + U_j(t^B)}
$$

Note that voters do not abstain, they either vote for $A$ or for $B$, so $\pi^A_j (t^A, t^B) + \pi^B_j (t^A, t^B) = 1$.

The higher the economic utility from platform $A$ the higher will be the probability that group $j$ (or a representative voter in group $j$ ) will vote for $A$. Parties are office-motivated.

They choose simultaneously the policy platform that maximizes $\pi^C(t^A, t^B) = \sum_j \pi^C_j(t^A, t^B)$ with $C=A, B$. Among the main findings of the probabilistic voting model we cite the following:

1.- Equilibrium existence and uniqueness. There exists equilibrium (a saddle point to $\pi^C(t^A, t^B)$) as long as $U_j(t^C)$ is quasiconcave on $t^C$. Note this is a multidimensional problem and a CW may not exist.
2.- Policy convergence. Both candidates face a similar problem $\pi^A(t^A, t^B)$ for $A$ and $\pi^B(t^A, t^B)$ for $B$. This implies that they both choose the same policy platform and the probability of winning equals $\frac{1}{2}$.

3.- The outcome of the political competition game is the social alternative that maximizes a Nash social welfare function (Theorem 1 of Coughlin and Nitzan, 1981).

Lindbeck and Weibull give a microfoundation to (generalize) Coughlin’s model. They introduced ideology. A voter may cast their ballot in favor of a candidate that gives him lower economic utility if the utility from the non-economic issue overweights the economic loss. Although voters may indeed vote deterministically the Lindbeck and Weibull model is called a probabilistic voting model because of the close link it has with Coughlin’s model.

We take the Lindbeck and Weibull approach here (section 6 explains the model in more detail) and discuss how our predictions change if we follow Coughlin.

### 2.2 Preferences

Voters are divided in 3 groups: poor (P), middle class (M) and rich (R). Population size is normalized to one. We assume the proportion of voters in group $P$ equals the proportion of voters in group $R$ which is $\alpha$, proportion of $M$ voters is then, $1-2\alpha$. Groups are differentiated by their marginal wage (ability) $w_j$, with $j=P, M, R$, such that $0 = w_P < w_M < w_R$.

Total income of an individual in group $j$ is $y_j = w_j l_j$, where $l_j$ is labor effort chosen by voter $j$. Consumption equals after-tax income, $c_j = y_j - T(y_j)$, with $T(y_j)$ being total tax payment by the $j$-voter.

We denote by $U_j (c_j, l_j)$ the utility of a member of group $j$ with consumption $c_j$, and labor supply $l_j \in [0, L]$, and assume that $U_j$ is increasing in consumption ($c_j$) and decreasing in labor ($c_j$) which can be seen as labor effort or hours worked per week, the last being an
imperfect measure of labor effort. For simplicity we assume the utility function is quasilinear in consumption, \( U_j = u(c_j + v(L - l_j)) \), \( U_j \) is well behaved: \( u' > 0, u'' < 0, v' > 0, \) and \( v''' < 0 \). This utility specification will allow us to make straightforward comparisons between the outcomes of the two probabilistic voting models. We assume that the elasticity of labor defined as: \( \varepsilon_j = \frac{\partial l_j}{\partial w_j} \) is decreasing in marginal wage.

### 2.3 The Tax Schedule

Each group \( j \) pays a marginal tax rate of \( t_j \) on income and receives a Lump-sum transfer \( G_j \). All income tax collected finances a public good level, \( G \) and lump-sum transfers \( G_M \) and \( G_R \), that favor group \( M \) and \( R \), respectively. The government budget condition is:

\[
G(t,G) = \sum_{j=P,R} \alpha_j t_j y_j + (1 - 2 \alpha) t_M y_M - (1 - 2 \alpha) G_M - \alpha G_R
\]

Provided our normalization of wages, where \( w_P = 0 \), the tax rate paid by group \( P \) is \( t_P = 0 \). Assume \( 0 \leq t_M, t_R \leq 1 \). This reduces the dimensionality of the economic platform to \( t = (t_M, t_R) \in T \), where \( T : [0,1] \times [0,1] \) is the set of possible income tax rates and \( G = G_M, G_R \geq 0 \). The tax schedule will be marginal rate progressive whenever income tax rate increase with income. This means, for our particular case, that \( t_R - t_M > 0 \). Further conditions should be given to guarantee that indeed \( y_P \leq y_M \leq y_R \). Given the disincentive effects from taxation we do not expect \( t_M \) or \( t_R \) to be larger than the income tax rates that maximize \( G \).

Given our tax schedule, from the government budget constraint balance condition, the political struggle takes place between two tax parameters: \( (t_M, t_R) \), and the lump-sum transfers \( (G_M, G_R) \), we can express the level of public good as:

\[
G(t,G) = (1 - 2 \alpha) t_M y_M + at_R y_R - (1 - 2 \alpha) G_M - \alpha G_R
\]

(1)

Since labor is endogenously supplied, the tax schedule should satisfy the following
conditions:
\[ y_R(t) \geq y_M(t) \geq y_P(t) \]  \hspace{1cm} (2)

In general, this condition is easier to satisfy the higher is the wage differential between groups R, M and M, P. Note that from preferences’ quasi-linear specification, labor supply does not depend on G. Once the winning platform is in place, voters choose their before tax income given the parameters of the tax function \((t, G)\), this is equivalent to choose their labor supply.

### 3 Optimal labor supply

For the income tax schedule \((t_j, t_R, G, G_M, G_R)\), the after-tax income is \(w_pl_p + G, (1 - t_M)w_Ml_M + G + G_M, (1 - t_R)w_Rl_R + G + G_R\), for the poor, middle class and rich voter, respectively. Given the tax parameters, voter-\(j\) decides over consumption and labor supply:

\[
\max_{c_j, l_j} U(c_j + v(L - l_j)) \quad \text{s.t.} \quad c_j \leq (1 - t_j)w_jl_j + G + G_j
\]

The optimal labor supply:

\[
((1 - t_j)w_j - v'(L - l_j))u'(x_j) = 0 \\
\Leftrightarrow l_j = L - h((1 - t_j)w_j)
\]

Where \(x_j = c_j + v(L - l_j)\) is consumption plus utility from leisure, and \(h = v'^{-1}\), with \(u'(x_j) > 0\); from the concavity of \(v\) we know \(h(.)\) is decreasing in its argument. From the quasi-linear specification of \(x_j\) there are no income effects, which implies that \(\frac{\partial l_j}{\partial t_j} < 0 \quad \left(\frac{\partial l_j}{\partial w_j} > 0\right)\) and \(\frac{\partial l_j}{\partial G} = 0\). Assume all voters supply strictly positive units of labor, that is \((1 - t_j)w_j - v'(L)\)
Given this optimal labor supply the tax schedule feasibility constraints in (2) can be rewritten as, \( L \)

\[
L(w_r - w_w) \geq w_r h((1-t_r)w_r) - w_r h((1-t_u)w_u)
\]

\[
L(w_u - w_r) \geq w_u h((1-t_u)w_u) - w_r h(w_r)
\]

The above feasibility conditions give an upper bound to \( t_M \) and \( t_R \); which are increasing in \((w_R - w_M)\) and \((w_M - w_P)\), respectively.

### 4 Preferences over tax schedules

Next we derive the group specific preferences over \((t, G)\), given \(G(t, G)\) specified in (1) and pre-tax incomes \(y_M = w_M(L - h(1-t_M)w_M)\) and \(y_R = w_R(L - h(1-t_R)w_R)\).

The indirect utility of voter \(P\),

\[
V_P(t_M, t_R, G_M, G_R) = u(G(t_M, t_R, G_M, G_R))
\]

Group \(P\) objective is to set \((t_M, t_R)\) that maximize the public good level \(G\), remember \(P\) does not pay income taxes. Assume \( \frac{\partial^2 l_j}{\partial t_j^2} \leq 0\), for \(j=M,R\). It is easy to show that \(G(t, G)\) will be concave in \((t_M, t_R)\) since \(\frac{\partial^2 G}{\partial t_M^2} = 0\), \(\frac{\partial^2 G}{\partial t_M^2} = (1-2\alpha)w_M\left(2\frac{\partial l_M}{\partial t_M} + t_M\frac{\partial^2 l_M}{\partial t_M^2}\right) < 0\), and

\[
\frac{\partial^2 G}{\partial t_R^2} = \alpha w_R\left(2\frac{\partial l_R}{\partial t_R} + t_R\frac{\partial^2 l_R}{\partial t_R^2}\right) < 0.
\]

Naturally \(P\) will choose \(G_M = G_R = 0\). The f.o.c. for a maximum,

\[
\tilde{t}_M : l_M + t_M \frac{\partial l_M}{\partial t_M} = 0
\]

\(\frac{\partial G}{\partial t_M} = 0\)

\(\frac{\partial G}{\partial t_R} = 0\)

(3)

(4)

\[b\] Utility functions satisfying this assumption and the assumption on the elasticity of labor supply: \(v(l_j) = - \frac{1}{2} (l_j)^2\), \(v(L- l_j) = \sqrt{(L-l_j)}\).
The preferred tax schedule of voter $P$ is the peak of the Laffer curve.

Rearranging terms in (3) and (4), the tax schedule maximizing $G$ satisfies,

$$\left| \epsilon_M - \epsilon_R \right| = 1$$

(5)

Where $\epsilon_j$ is the elasticity of labor supply to changes in the tax rate $t_j$ with $j=M,R$.

$$\left| \epsilon_j \right| = \left| \frac{\partial l_j}{\partial t_j l_j} \right|.$$  

Whether the $P$-voter preferred tax schedule would be proportional, marginal rate progressive or regressive depends upon the specific utility function. For $\frac{\partial \epsilon_j}{\partial t_j} \leq 0$, $\epsilon_j$ will be decreasing in $t_j$. After some computations $\epsilon_j = -\frac{t_j}{l_j} \epsilon_j$. By assumption $\epsilon_j$ is decreasing (specifically non increasing) in $w_j$ which implies that $\epsilon_j$ will be increasing in $w_j$.

These properties altogether ensure that the tax schedule maximizing the level of public good, determined by (5), can not be marginal-rate regressive. To show this, consider $\epsilon_j$ at the proportional tax rate $t_M = t_R = t$, since $\epsilon_j$ is increasing in $w_j$, then $\epsilon_M(t) \leq \epsilon_R(t)$. If we increase $t_M$ such that $t_M > t_R = t$, being $\epsilon_j$ decreasing in $t_j$ then, necessarily $\epsilon_M(t_M) < \epsilon_M(t) \leq \epsilon_R(t)$. This proves that, under our assumptions on preferences, the preferred tax schedule of voter $P$ is either proportional ($t_M = t_R$) or progressive ($t_M < t_R$).

Group $M$ pays income taxes at rate $t_M$ and receives the lump-sum transfer $G_M + G$. Thus, group $M$ preferred income tax minimizes his tax burden. The indirect utility of a voter $M$,

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$^c$ Given that there are only substitution effects from taxation and by assumption the derivative of $l_j$ with respect to $t_j$ is decreasing in $t_j$, $\epsilon_j$ will be decreasing in $t_j$: 

$$\frac{\partial \epsilon_j}{\partial t_j} = \frac{\partial l_j}{\partial t_j} + \frac{\partial l_j}{\partial t_j} \left( \frac{t_j}{l_j} \right) < 0.$$
The indirect utility \( V_M(t, G) \) reaches a (global) maximum at \((t_M, t_R) = (0, \hat{t}_R)\) and

\[
G_M = \frac{\alpha}{1 - 2\alpha} \hat{t}_R v_R(t_R).
\]

For all \( t_M, t_R \) belonging to \( T \). Remember that \( \hat{t}_R \) maximizes \( G(t, G) \) for a given \( t_M \).

Note that such tax schedule is marginal rate progressive since \( t_R - t_M = \hat{t}_R > 0 \).

Group \( R \) pays income taxes at rate \( t_R \) and receives the lump-sum transfer \( G + G_R \). Thus group \( R \) preferred income tax minimizes his tax burden. The indirect utility of a voter \( R \),

\[
V_R(t_M, t_R, G_M, G_R) = u((1 - t_M)wM + G(t_M, t_R, G_M, G_R) + G_R)
\]

Her utility is maximized under a regressive tax schedule: \( t_M = \hat{t}_M \) and \( t_R = 0 \).

Remember that \( \hat{t}_M \) maximizes \( G(t, G) \) for a given \( t_R \).

The following picture shows the different voters’ bliss-points.

[Figure 1 here]

It should be noted that no CW winner exists in our voting game, as we can see in figure one. Any point in rectangle 0RPM can be defeated by a coalition of two groups. The shaded areas in Figure 1 represent the alternatives that can defeat alternative "o", which can be defeated by other alternatives generating a cycle (the voting paradox).

Next section studies conditions under which only progressive taxes emerges in equilibrium in the probabilistic voting model.

\[d\] The plot was made for the particular utility function \( U_j = c_j - \frac{1}{2} t_j^2 \). For this utility function \( \hat{t}_M = \hat{t}_R = \frac{1}{2} \).
5 Political Competition

We consider electoral competition between two office-motivated parties, $A$ and $B$. They differ in their fixed ideology position and may differ in the income tax schedule they announce. Parties commit themselves to the platform announced.

We have a continuum of voters in each group differing in their ideological position, measured as their relative preference from one party over the other. In order to combine the economic and ideological side of voters' utility we assume that a voter $i$ in group $j$ will vote for party $A$ if the extra "economic" utility he gets from the $A$’s platform exceeds his ideological preference for $B$ relative to $A$. We may capture preferences over parties through parameter $\sigma_{ij}$, which is the location of voter $i$ in group $j$ along the real line. A positive (negative) $\sigma_{ij}$ means that $i$ in group $j$ prefers $B$ ($A$) to $A$ ($B$) for the same platform announced. Voters with $\sigma_{ij}$ around zero are ideological neutral; they mainly evaluate the economic benefit they receive from the different platforms proposed by parties.

The utility of a $ij$-voter is simply $V_j(t^d, G^d)$ if party $A$ wins and it is $V_j(t^B, G^B) + \sigma_{ij}$ otherwise.

A voter $i$ in group $j$ will vote for $A$ if:

$$V_j(t^d, G^d) > V_j(t^B, G^B) + \sigma_{ij}$$  \hspace{1cm} (6)

We assume that $\sigma_{ij}$ has group-specific cumulative distribution function $F_j$ with density $f_j$ with support on the real line. The density function around zero gives us the proportion of ideologically neutral voters within each group. We next introduce conditions that guarantee existence of a unique pure strategy Nash equilibrium of the electoral game. Such
conditions, from Lindbeck and Weibull (1987), Enelow and Hinich (1989) and Coughlin (1992), were unified by Banks and Duggan (2004). Apart from those conditions, we need to add an additional one since in our model $F_j$ is not independent of $j$.

**Conditions On $F_j$ and aggregate $V_j$:**

1. **C1.** $F_j$ is continuous and strictly increasing.

2. **C2.** Aggregate concavity holds, for any $t^C$, $\pi^C(t^A, t^B)$ is strictly concave on $t^C$, $C = A, B$:

$$\pi^A(t^A, G^A ; t^B, G^B) = \sum_{j=P,M,R} n_j F_j \left( V_j(t^A, G^A) - V_j(t^B, G^B) \right)$$

$$\pi^B(t^A, G^A ; t^B, G^B) = 1 - \sum_{j=P,M,R} n_j F_j \left( V_j(t^A, G^A) - V_j(t^B, G^B) \right)$$

Where $n_j$ is the proportion of voters in group $j$.

3. **C3.** Laussel and Le Breton (2002). This condition guarantees that at the political equilibrium of this game there is no profitable deviation.

$$\forall j \ f_j \text{ is symmetric around zero and } f_j(0) > 0.$$

We define the swing voter in group $j$ as the voter that is indifferent between party $A$ and party $B$ given the policies announced. Let's call it $\sigma_j$, from (6), $\sigma_j = V(t^A, G^A) - V(t^B, G^B)$. Voters in group $j$ with an ideological parameter, $\sigma_{ij}$, smaller (or higher) than $\sigma_j$ will vote for party $A$ (respectively $B$). We assume there is no abstention.

The total share of votes of party $A$ in group $j$, $\pi^A_j(t^A, G^A, t^B, G^B)$ is:

$$\pi^A_j(t^A, G^A ; t^B, G^B) = n_j \ Pr(\sigma_j < \sigma_{ij}) = n_j F_j(\sigma_j).$$

Total voting share of party $A$ is:

$$\pi^A(t^A, G^A ; t^B, G^B) = \alpha F_p(\sigma_p) + (1 - 2\alpha) F_M(\sigma_M) + \alpha F_s(\sigma_s)$$

The probability of winning is an increasing function of the voting share. For simplicity, we
assume the probability of winning equals aggregate voting share. The main reason why we
make such assumption is that it allows us to compare the outcome of the Coughlin game
(where parties maximize the probability of winning) with the outcome of the Lindbeck and
Weibull game (where parties maximize their expected voting share).
The game presented above will be called LW from Lindbeck and Weibull. We assume for
such a game that \( u(x_j) \) is logarithmic with \( x_j = c_j + \nu(L-I) \).
Without loss of generality we write the probability of winning of party \( A \) simply as
\[ \pi(t, G; t^A, G^A) \]
where \((t, G)\) is the platform chosen by party \( A \), and \((t^B, G^B)\) the platform
chosen by \( B \).

**Lemma 1** Assume conditions C1, C2 and C3 are satisfied. The bliss points of group P, M
and R are never part of a political equilibrium.

**Proof** a) The bliss point of \( P \) is not an equilibrium. If it was an equilibrium then for a
given \((t^A, G^A)\),
\[ \frac{\partial \pi(t, G; t^A, G^A)}{\partial t_M} dt_M + \frac{\partial \pi(t, G; t^A, G^A)}{\partial t_R} dt_R \bigg|_{t_M=t^M, t_R=t^R} = 0 \]
for \( dt_M, dt_R < 0 \). We next show that \( \pi'(t^A, G^A, t^B, G^B) \) actually increases as we move from \((\hat{t}_M, \hat{t}_R)\) to lower tax
rates.
\[ \frac{\partial \pi(t, G; t^A, G^A)}{\partial t_M} \bigg|_{t_M=i, i_R=i} = (1 - 2\alpha) f_M \left( \sigma_M \right) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) \bigg|_{t_M=i} < 0 \]
\[ \frac{\partial \pi(t^A, G^A; t^B, G^B)}{\partial t_M} \bigg|_{t_M=i, i_R=i} = (1 - 2\alpha) f_M \left( \sigma_M \right) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) \bigg|_{t_M=i} < 0 \]
\[ \frac{\partial \pi(t^A, G^A; t^B, G^B)}{\partial t_R} \bigg|_{t_M=i, i_R=i} = \alpha f_R \left( \sigma_R \right) \left( \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) \bigg|_{t_R=i} < 0 \]
This proves that the preferred income tax of the \( P \) group can not be part of an equilibrium.

b) The bliss point of \( M \) is not an equilibrium.
\[
\frac{\partial \pi(t^A, G^A; t^B, G^B)}{\partial t_M} \bigg|_{t_M=0, t_R=0} = (1 - 2\alpha) f_t M \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) \bigg|_{t_M=0} > 0
\]

\[
\frac{\partial \pi(t^A, G^A; t^B, G^B)}{\partial t_R} \bigg|_{t_M=0, t_R=0} = \alpha f_t R \left( \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) \bigg|_{t_R=0} < 0
\]

This proves that an increase in \(t_M\) and a decrease in \(t_R\), from the preferred platform of voter M, improves party A chances of winning. Then, the preferred income tax of the M group can not be part of equilibrium.

c) The bliss-point of R is not an equilibrium.

\[
\frac{\partial \pi(t^A, G^A; t^B, G^B)}{\partial t_M} \bigg|_{t_M=0, t_R=0} = (1 - 2\alpha) f_t M \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) \bigg|_{t_M=0} < 0
\]

\[
\frac{\partial \pi(t^A, G^A; t^B, G^B)}{\partial t_R} \bigg|_{t_M=0, t_R=0} = \alpha f_t R \left( \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) \bigg|_{t_R=0} > 0
\]

This proves that the preferred income tax of the R group can not be part of the equilibrium.

Moreover for \(f_R\) sufficiently small \(G_R=0\) in equilibrium.

The above proposition proves that under this political process the outcome will never correspond with the ideal income tax schedule of some group, because the probabilistic model implies some compromise between the voters. The fact that the probabilistic model in a multidimensional space picks a policy that is different from the ideal of some voter was already stressed in Casamatta, Cremer and Pestieau (2006). Indeed the equilibrium outcome in our setting is the outcome of a Nash bargaining between groups of voters for a linear utility, \(u(x) = x\), with the political (bargaining) power of group P, M, and R given by \(f_p(0)\), \((1-2\alpha) f_M(0)\) and \(f_R(0)\), respectively, with the disagreement utility set at zero for every group.
Proposition 1. Assume C1, C2 and C3 are satisfied. Assume that for all \( t_R \leq \hat{t}_R \) and \( t_M \leq \hat{t}_M \) the feasibility constraints in (2) are satisfied. There exists a unique interior equilibrium. By symmetry of the game, at this equilibrium we have policy coincidence, \( \mathbf{t}^A = \mathbf{t}^B = \mathbf{t} \) with \( t_M < \hat{t}_M \) and \( t_R < \hat{t}_R \).

Proof. Uniqueness comes from the fact that we maximize a strictly concave function (C2) under a convex set \( T \). By symmetry of the model if \( (\mathbf{t}^A, \mathbf{t}^B) \) is an equilibrium so it is \( (\mathbf{t}^B, \mathbf{t}^A) \) from uniqueness necessarily \( \mathbf{t}^A = \mathbf{t}^B = \mathbf{t} \). If (2) is not binding, parties choose \( \mathbf{t}^C \) that maximizes a weighted sum of voters utilities (Lindbeck and Weibull, 1987). Then, any tax \( t_j > \hat{t}_j \) with \( j = M, R \); will not be played in equilibrium since it is Pareto dominated. From Lemma 1 we know that the equilibrium tax schedule is different from \( (\hat{t}_M, \hat{t}_R) \), the bliss-point of group \( P \). Note that feasibility constraints in (2) can be omitted (will not be binding) as long as \( \tilde{t}_R \geq \hat{t}_R \) and \( \tilde{t}_M \geq \hat{t}_M \), where \( \tilde{t}_R \) is the lowest possible \( t_R \) satisfying (2) (remember that \( y_j \) is decreasing in \( t_j \)): \( y_R(\tilde{t}_R) - y_M(0) = 0 \). Similarly for \( t_M \): \( y_M(\tilde{t}_M) - y_P = 0 \). Moreover \( \tilde{t}_R, \tilde{t}_M \) are higher the higher is \( w_R \) and the lower is \( w_P \).

The outcome of the LW game maximizes the following social welfare function:

\[
S^{LW}(\mathbf{t}, \mathbf{G}) = \alpha f_P V_P (\mathbf{t}, \mathbf{G}) + (1 - 2\alpha) f_M V_M (\mathbf{t}, \mathbf{G}) + \alpha f_R V_R (\mathbf{t}, \mathbf{G})
\]

where \( f_j = f_j(0), j = P, M, R \).

Since we are interested in tax progressivity next we develop conditions under which a progressive income tax emerges as the outcome of the LW game. Note that our assumption on the elasticity of labor supply (we assume that \( \varepsilon_l \) is decreasing in \( w \)) facilitates the implementation of a progressive income tax since the labor response of the \( R \)-group to changes on the marginal tax rate they pay is lower than that of the \( M \) group. Moreover the
decreasing marginal utility of consumption (net of labor disincentives) facilitates the emergence of a progressive income tax schedule by increasing the political power of groups $P$ and $M$ compared to that of group $R$. In other words, the marginal utility loss from an increase in the tax rate $t_R$ is lower for group $R$ than it would be a proportional increase in $t_M$ for $M$’s utility, which makes group $M$ more sensitive to changes on $t_M$. Despite all this a proportional or even marginal regressive income tax may arise in equilibrium if the proportion of ideologically neutral voters, $f_R$ is sufficiently large.

The equilibrium income tax satisfies the following first order conditions,

$$
\Phi(t_M, t_R)(1 - |e_M(t_M)|)x_M(t_M, t_R) - f_M = 0
$$
$$
\Phi(t_M, t_R)(1 - |e_R(t_R)|)x_R(t_M, t_R) - f_R = 0
$$

where $\Phi(t_M, t_R) = \frac{\alpha_x}{x_M(t_M, t_R)} + \frac{(1-2\alpha)\mu_M}{x_M(t_M, t_R)} + \frac{\alpha_x}{x_R(t_M, t_R)}$.

The tax schedule will be marginal rate progressive if at the proportional tax schedule there is a profitable deviation to a more progressive schedule (higher $t_R$ or/and lower $t_M$). From uniqueness this would imply that the equilibrium income tax schedule is not proportional, nor regressive since there would not be a profitable deviation from moving toward a regressive tax (lower $t_R$). The condition is,

$$
\frac{(1 - |e_M(t)|)x_M(t, t)}{(1 - |e_R(t)|)x_R(t, t)} < \frac{f_M}{f_R}
$$

Since $e_j$ is increasing in $w_j$ the expression $\frac{(1-2\alpha)\mu_M}{\mu_M(t)} < 1$. Given that, $x_j$ is increasing in $w_j$ ($\frac{\partial x_j}{\partial w_j} = (1-t_j)l_j > 0$), which implies that $\frac{x_M(t, t)}{x_R(t, t)} < 1$. If $f_M \geq f_R$, only marginal progressive taxes emerge in equilibrium (note that this is stronger than needed).

**Proposition 2** The equilibrium income tax is marginal progressive as long as the
inequality below holds,
\[
\frac{(1-|\varepsilon_M(t)|)x_M(t,t)}{(1-|\varepsilon_R(t)|)x_R(t,t)} < \frac{f_M}{f_R}
\]  \tag{7}

**Proof** From the proportional tax a progressive tax is a profitable deviation for Party A if,
\[
[(\Phi(t,t)(1-|\varepsilon_M(t)|)x_M(t,t) - f_M)dt_M + (\Phi(t,t)(1-|\varepsilon_R(t)|)x_R(t,t) - f_R)dt_R] > 0
\]
for 
\(dt_M<0\) and \(dt_R>0\). Dividing both sides by the RHS of the expression in brackets and rearranging terms we find condition (7) for progressivity in the LW game.

We now follow the approach of Coughlin (1992). From Coughlin and Nitzan (1981) we know that the outcome of the electoral competition game is the social alternative that maximizes a Nash social welfare function. For simplicity we assume that
\[
V_j(t_M,t_R) = x_j(t_M,t_R).
\]

The party's objective function is then,
\[
S^{CN}(t,G) = \alpha \ln x_p(t,G) + (1-2\alpha) \ln x_M(t,G) + \alpha \ln x_R(t,G)
\]

Note that the equilibrium outcome of this game is the result of a Nash bargaining between the three groups of voters with the political (bargaining) power of group P, M and R given by \(\alpha\), \((1-\alpha)\) and \(\alpha\), respectively, and the disagreement utility set at zero for every group. In this game the political power of the poor and the rich are the same the first prefers a progressive (or proportional) income tax and the last a regressive tax, while voter M unambiguously prefers a progressive income tax, hence only marginally-rate progressive taxes emerge in equilibrium.

**Proposition 3** *If the elasticity of labor supply decreases with wage, only marginal rate progressive taxes emerge in equilibrium.*

**Proof** Note that the CN game is equivalent to the LW game for \(F_j\) independent of \(j\) (this
was previously stressed by Banks et al., 2004). In such a case $f_j = f$, i.e. $f_M = f_R$. As proved in Proposition 2, this is a sufficient condition for marginal progressive taxes.

When voters choose a candidate (party) probabilistically the best response for both candidates is to announce a marginal rate progressive tax schedule. Because, when competing for elections, parties try to attract swing voters, whose probability to vote for the party increases a lot in response to a marginal increase in their consumption. The probability that a group vote for a given party, say $A$, is concave in their consumption (being $\ln V_j$ a proxy for the probability of voting for the party). Then, voters in group $P$ and $M$ are more attractive than voters in group $R$, since they increase faster the probability to vote for the party that benefits them. Under our assumption of decreasing elasticity of labor on wages, the preferred tax schedule of $P$ is either proportional or progressive. This guarantees that a move from (proportionality) regressivity toward progressivity is profitable. It captures more swing voters, since the marginal gain in increasing consumption for group $M$ is higher than the marginal loss caused to group $R$.

5.1 Comparative Statics

We wonder at this point how the degree of progressivity changes as a result of a change in the parameters of the model. Assume (2) is satisfied at the solution of $S^{CN}$: $(t', G')$.

**Proposition 4** 1. In the CN game an increase in population polarization, measured by $\alpha$, decreases the progressivity degree.

2. An increase in $f_M$ ($f_R$) decreases the equilibrium tax rate $t_M$ ($t_R$). Public good level is increasing in $f_P$ for the LW game.
Proof At the equilibrium tax schedule \((t^*_M, t^*_R)\),

\[
\frac{\partial \pi}{\partial t_M} = \alpha \left( \frac{1}{x_R} \frac{\partial x_P}{\partial t_M} + \frac{1}{x_M} \frac{\partial x_R}{\partial t_M} \right) + (1 - 2\alpha) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) = 0 \tag{8}
\]

\[
\frac{\partial \pi}{\partial t_R} = \alpha \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_R} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) + (1 - 2\alpha) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_R} \right) = 0 \tag{9}
\]

We know that for \(t^*_M > 0\), \(\frac{\partial x_M}{\partial t_M} < 0\). Then, necessarily \(\left(\frac{1}{x_P} \frac{\partial x_P}{\partial t_M} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_M}\right) > 0\). While for \(t^*_R < \hat{t}_R\), \(\frac{\partial x_M}{\partial t_M} > 0\). Then, necessarily \(\left(\frac{1}{x_P} \frac{\partial x_P}{\partial t_M} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_M}\right) < 0\). Consider now a different economy with \(\alpha' > \alpha\). At \((t^*_M, t^*_R)\) it can be easily showed that a higher \(\alpha\) gives a higher weight to the negative part of the f.o.c. in (9) and a lower weight to the positive part \(\frac{\partial x_M}{\partial t_R}\).

Then at \((t^*_M, t^*_R)\),

\[
\frac{\partial \pi}{\partial t_R} = \alpha' \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_R} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) + (1 - 2\alpha) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_R} \right) < 0
\]

From concavity of \(\pi(\cdot)\), this implies that the equilibrium tax rate \(t_R\) at the economy \(\alpha'\) is lower than \(t^*_R(\alpha)\), i.e. \(t^*_R(\alpha') < t^*_R(\alpha)\). Likewise, at \((t^*_M, t^*_R)\) \(\frac{\partial \pi}{\partial t_M} > 0\) for \(\alpha' > \alpha\),

\[
\frac{\partial \pi}{\partial t_M} = \alpha' \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_M} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_M} \right) + (1 - 2\alpha) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) > 0
\]

From concavity of \(\pi(\cdot)\), this implies that the equilibrium tax rate \(t_M\) at the economy \(\alpha'\) is higher than \(t^*_M(\alpha)\), i.e. \(t^*_M(\alpha') > t^*_M(\alpha)\). Finally,

\[
t^*_R(\alpha') < t^*_R(\alpha) \quad \text{and} \quad t^*_M(\alpha') > t^*_M(\alpha)
\]

\[\Rightarrow t^*_R(\alpha') - t^*_M(\alpha') < t^*_R(\alpha) - t^*_M(\alpha)\]
The progressivity degree is lower the higher is population polarization.

2. In the LW model applying the implicit function theorem to the first order conditions we study how \( t_M \) changes in response to a change in \( f_M \).

\[
\frac{\partial t_M}{\partial f_M} = \left( \frac{\partial \Phi}{\partial f_M} (1 - |\varepsilon_M(t_M)|) x_M(t, G) - 1 \right) / D
\]

where \( D = \left( \frac{\partial \Phi}{\partial f_M} (1 - |\varepsilon_M(t_M)|) x_M(t, G) + \Phi \frac{\partial \pi}{\partial f_M} x_M(t, G) + \Phi(1 - |\varepsilon_M(t_M)|) \frac{\partial \pi}{\partial f_M} \right) < 0 \), by concavity of \( \pi(t, G) \) on \( T \). Substituting \( sgn \) in the above equation:

\[
sgn \left( \frac{\partial t_M}{\partial f_M} \right) = sgn \left( (1 - 2\alpha)(1 - |\varepsilon_M(t_M)|) - 1 \right) < 0.
\]

Similarly for \( t_R \),

\[
\frac{\partial t_R}{\partial f_R} = \left( \frac{\partial \Phi}{\partial f_R} (1 - |\varepsilon_R(t_R)|) x_R(t_M, t_R) - 1 \right) / D,
\]

Then, \( sgn \left( \frac{\partial t_R}{\partial f_R} \right) = sgn(\alpha(1 - |\varepsilon_R(t_R)|) - 1) < 0. \)

The degree of progressivity decreases with \( f_R \), since \( \frac{\partial \Phi}{\partial f_R} = \frac{1-2\alpha}{x_M(t_M, t_R)} > 0 \) and more generally \( sgn \left( \frac{\partial f_R}{\partial f_j} \right) = sgn \left( \frac{\partial \Phi}{\partial f_j} \right) > 0 \), \( j = P, R \). Finally the lump-sum transfer level \( G(t_M, t_R) \) is increasing in \( f_P \), the political power of the group whose preferred tax schedule maximizes \( G \), note that since \( G_M \) and \( G_R \) does not benefit group \( P \), both decreases with \( f_P \).

\[
\frac{\partial G}{\partial f_P} = \frac{\partial G}{\partial t_M} \frac{\partial t_M}{\partial f_P} + \frac{\partial G}{\partial t_R} \frac{\partial t_R}{\partial f_P} + \frac{\partial G}{\partial G_M} \frac{\partial G_M}{\partial f_P} + \frac{\partial G}{\partial G_R} \frac{\partial G_R}{\partial f_P} > 0
\]

If we think of \( (t_M^*, t_R^*) \) as the solution to a bargaining among groups \( P, M \) and \( R \) with the disagreement option settled at zero utility, and \( \alpha f_P, (1 - 2\alpha) f_M \) and \( \alpha f_R \) as the bargaining (political) power of \( P, M \) and \( R \), respectively. The previous result states that the degree of progressivity decreases with the political power of group \( M \), the group whose preferred tax is maximal progressivity. In CN the size of a group measures his political power, thus
regardless of how population is distributed among groups, tax progressivity increases as the size of group $M$ increases (or population polarization decreases). This is the case in both models. Finally in LW the lump-sum transfer or public good level, $G$, is unambiguously increasing in the political power of group $P$.

6 Conclusions

We wanted to show that despite there is only substitution effects from taxation, only marginal rate progressive taxes will emerge as the political equilibrium for the CN game.

Our assumption on the elasticity of labor supply (that $\varepsilon_l$ is decreasing in marginal wage) is crucial for our result. It facilitates the implementation of marginal progressive taxes in both models (CN and LW) with respect to the fixed or exogenous income case. Indeed the condition on Proposition 1 is stronger to satisfy at $\varepsilon_l = 0$: $\frac{\frac{x_{\Delta}(r, t)}{x_{\Delta}(t, r)}}{\frac{x_{\Delta}(t, r)}{x_{\Delta}(r, t)}} < \frac{f_{\Delta}}{f_{\Delta}}$ (condition in Proposition 1 for progressivity at the fixed income case). In this context labor disincentives gives makes the richer group cheaper to tax than the middle class.

Since the CN and the LW political equilibrium could be understood as the result of the bargaining among groups, tax progression is decreasing in population polarization in CN, which is equivalent to the size (bargaining power) of the $P$ and $R$ groups. For analogous reasons tax progression will be increasing in the political power of the middle class, $(1-2\alpha)f_M$, for the LW game. A larger degree of marginal progressivity is expected in societies with a stronger middle class.
Bibliographical references


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