Strategic Pricing, Auctions and Fragmented Market under Universal Service Obligation

Ramiro Losada†

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Abstract

We develop a simple multi-market model with an oligopolistic (profitable) area and entry auctions for the unprofitable area. We examine how the Universal Service Obligations and the possibility of subdividing this areas affect pricing and Social Welfare. We find that to allow entry in the unprofitable area gives a lower price and a lower equilibrium bids compering with the case of no entry. Moreover, if we restrict the unprofitable area to be served by the entrants, we are in the Social Welfare maximizing situation.

Keywords: Auctions, Pricing, Universal Service Obligation.

JEL Classification: D4, L1, L5, L96.
1 Introduction

There are many industries that have price constraints across market segments. Very often, these price constraints are due to Universal Service Obligations and they arise when the difference between the price constraint and unconstraint are deemed unacceptable by policymakers. Typical examples of industries under Universal Service Obligation are postal delivery, railway transport or telecommunications. Liberalization in many regulatory environments, however, has exposed profitable markets to entry and competition. As discussed by Laffont and Tirole and others, liberalization has raised important questions regarding the coexistence of price constraints across markets, competition, and the goal of Universal Service, as many markets and segments are inherently unprofitable on a stand-alone basis.\(^1\)

If we focus on the literature about multi-market competition under cross-market competition under cross-market price restriction. Armstrong and Vickers (1993) analyze the effects of price discrimination when an incumbent has to compete with an entrant (price taking) in a profitable market and the incumbent serves other market as well. They prove that the impossibility of price discrimination across markets, which is part of the Universal Service requirements, causes the incumbent to be less aggressive when there is entry.

In this environment, an important issue treated by Anton et al. (2002), is the firm selection for a second and unprofitable market and the strategic linkage between this market and the profitable market where there is competition. In particular, they examine the use of an auction to determine which firm will supply the unprofitable market. As Armstrong (2000) and Laffont and Tirole (2000) pointed out, there is significant policy interest in the potential for awarding the right to serve markets via auctions in which bidders compete on requested subsidy levels. They analyze the strategic implications of Universal Service Obligations and related cross-market restrictions with simple model involving oligopolistic competition and two markets. In this context, they find that Universal Service Obligation create a strategic link between the profitable and unprofitable market. This link arises because a firm that supplies both markets would like to set a price in the unprofitable area in excess of the price in the competitive area. this makes these firms softer competitors in the profitable market.

\(^1\)According to Laffont and Tirole (2000): "Universal Service is knotty and explosive problem. It has been (or will be) a central issue in the political debate surrounding regulatory reform in all network industries and in most countries."
These authors also study how the presence of outside firms in the unprofitable market auction affects bidding and the resulting market structure.\(^2\) In this case the critical feature is that an outside firm cannot directly affect the price and therefore, when an outside firm wins the auction, the strategic link between markets disappears. This has important implications for bidding incentives as the identity of the winning firm now matters to an incumbent profitable market firm when it loses the auction.

This paper also treat the issue of firm selection for the unprofitable market and the strategic links between the profitable and unprofitable markets but with two important differences. First, to consider that the profitable market is subdivided in separated markets. This scenario is interesting because it picks up industries with this structure, e.g. telecommunications in Spain or in the U.S., and it may be a way of improving Social Welfare from what is studied in Anton et al. (2002). Second, the firm selection is made through a choice of an auction made by the policymakers. An important issue, and perhaps the most important, given that the profitable area is subdivided, is how the unprofitable area should be divided to fit the obligations of the Universal Service as the equality of price across all the areas, and at the same time, to try to maximize welfare, paying attention at how the division of the unprofitable area affects the final price and the subsidies required by the firms that serve the unprofitable market.

We also analyze how affects the introduction of outside firms in the unprofitable through two different type of auctions, and which is better from a welfare maximizing point of view. In auction type I, there are markets auctioned among just inside firms and areas auctioned just among outside firms. In auction type II the areas are just auctioned among outside firms. The reader can think why in any case the outsiders compete with the inside firms for the submarkets in the unprofitable market. The reason is that due to the constraint of a common price, there are just three types of equilibria. There are inside and outside firms operating which is picked up in auction type I, there are just outsiders which is picked up in auction type II and there are just inside firms what is picked up by the case of no entry.

Our main results say that it is welfare maximizing to allow outsiders to enter to bid for the unprofitable market using the auction type II. With this decision, on one hand, the consumers surplus is the maximum because the outsiders have no strategic links with the profitable market and moreover

\(^2\)An outside firm is a firm that does not serve in the profitable area.
when the unprofitable market is small, the subsidy is the smallest as well. On the other hand, the subsidy could be lower when the unprofitable market is big using auction type I, but the saving on subsidies never compensates the loss in consumers surplus due to a higher price. Furthermore, this result is independent of how we have divided the unprofitable market, because again, the outside firms do not have strategic links with the profitable market.

The idea of this paper is closely related to the more general issue of multimarket oligopoly, Bullow et al. (1985). Since cross-market price constraints can make softer competitors, this paper is also related to DeGraba (1987). The focus is different, however, as DeGraba shows that once firms become softer (price) competitor they may adjust their locations and, as a result, prices may decrease when a cross-market price constraint is present. In addition, he does not examine entry, a primary consideration in our analysis. There are two papers that are close to the present analysis. Choné et al. (2002) and Valletti et al. (2002) also consider Universal Service Obligations and their implications on firms’ strategy space. But their emphasis is more in funding mechanism, Choné et al. (2000), and in investigating the implications of the role of price and coverage constraint for firms and Welfare, Valletti et al. (2002).

The basic model is presented in Section 2. In Section 3, the basic model is analyzed. Section 4 discusses Welfare effects. We conclude in Section 5.

2 The model

The model is based in the model of Anton et al. (2002). There is a country that has two differentiated areas, U (urban) and R (rural). Demand in the U area is $D^U(p) = 1 - p$ and in the R area $D^R(p) = b(1 - p)$, where $p$ is the market price and $b > 0$. Thus, while both markets have a common price intercept of $p = 1$, the slope coefficient of $b$ allows rural demand to be smaller or larger than urban demand. In many situations, we expect the rural area to be smaller.

This paper makes an important difference from the paper of Anton et al. (2002). They consider that a duopoly operates in the U area while in the R area just a firm operates that could be a firm from the duopoly of the U

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3Our paper is related to the problem of introducing competition into regulated market (see Laffont and Tirole (1993) or De Fraja (1997), who also provide additional references).

4I use areas as synonym of market.
area or an outside firm. We consider, at first, that the U area is divided in $N$ parts that can be equals or not, in any of these parts there is a duopoly. The R area is also divided but in $M$ parts that can be equals or not as well, These areas are operated by firms from the duopolies or by outside firms. $N$ can be apriori greater, lower or equal than $M$.

The fixed costs of any given in any part of the U area is $\delta_i F^U$, $i = 1, \ldots, N$ $\sum^N_1 \delta_i = 1$ where $F^U > 0$. The fixed costs in any part of the R area is $\gamma_j F^R$, $j = 1, \ldots, M$ $\sum^M_1 \gamma_j = 1$ where $F^R > 0$. There is a constant marginal cost $c \geq 0$, that is the same for all the firms in both areas. Naturally, we assume that $c < 1$ so that, ignoring fixed costs, it is always profitable (and efficient) to supply some amount to each area.\(^5\)

### 2.1 The Game

We consider a simple complete information game with the following timing.

1. The government decides in how many parts each area is divided. Or to put it in another way, they decide the quantities of $M$ and $N$ with the constraint that the equilibrium prices in the different proportion of the urban area must be equal.\(^6\)

2. Firms $A_i$ and $B_i$ of the duopoly $i = 1, \ldots, N$ choose bids for just one of the parts $j = 1, \ldots, M$ in which the rural area is divided. If $M > N$ then, the part of the rural area $N + 1, \ldots, M$ are auctioned among outsiders that do not operate in the urban area. If $M < N$ then, just $N - M$ duopolies of the urban area can operate in the rural area. The bids of any firm represent lump-sum subsidies that the firms ask from the government in order to serve a portion of area R.

3. In any area in which the rural area is divided $j = 1, \ldots, M$ the lowest bidder (smaller subsidy required) wins, receives a subsidy equal to the

\(^5\)The variable costs in industries that support Universal Service Obligation usually tend to 0 as telecoms or water. Another important issue is that if firms have different costs the results presented here still hold as far as the size of the rural market allows the inefficient firms to compete in the U area.

\(^6\)Governments up to now do not allow differences in prices in profitable areas. For example in Spain, the government would not allow a difference in prices between Barcelona and Madrid.
the winning bid, incurs the fixed costs $\gamma_j F^R$ and becomes a monopolist in area R. ”Ties” among symmetric bidders are resolved by a coin toss.\footnote{In a complete information setting, auction equilibria typically involve a tie either because bidders are symmetric or because the bidder with the strongest strategic position bids so that the next strongest bidder is indifferent between losing and winning. Our tie rule follows the literature (see e.g., Milgrom (1987)).}

4. Firms $A_i$ and $B_i$ of all duopolies $i = 1, \ldots, N$ choose quantities $q_{A_i}$ and $q_{B_i}$ for the urban area they are in charge. The prices in any of the parts are $p^U_i = (1 - (q_{A_i} - q_{B_i})/\alpha_i), i = 1, \ldots, N$, where $\alpha_i$ measures the portion of the urban area duopoly $i$ is in charged, $\sum_i N \alpha_i$.\footnote{We denote by $\beta_j$ the different portion in which R area is divided, $\sum_j M \beta_j = 1$} The prices across the different portion must be equal.\footnote{See step 1 of the timing of the game.}

5. The different monopolists of the rural area can choose a price that cannot exceed the price determined in the U area, that is $p^R_j \leq p^U \forall j = 1, \ldots, M$.\footnote{This is one of the constraints that Universal Service Obligation impose to the firms that serve in the rural area.}

6. Each firm’s payoff is the sum of its profits in the two markets, including any subsidies.

We solve for a subgame perfect equilibrium of this game focusing on pure strategy equilibria in the bidding game.\footnote{There are some remarks about the model that are very much the same as in the model of Anton et al. (2002). First, the adopted time of events is the one that makes the cross-market constraint operate in a natural way. An alternative sequencing would be to have the firm that operates in both markets choose the quantities it supplies in each market at the same time that the other firm chooses its U market quantity. However, this would create the problem of how to impose the price constraint in the R area. Second, the multi-market firms should not be viewed as a price-takers in the R area. Given the cross-market price constraint, the firm is free to set any price in the R area up to the ceiling. More importantly, the ceiling is endogenous with respect to the firms’ actions: the multi-market firms can and do not adjust its U market choices to raise the ceiling price for the R market. Finally, the Cournot structure for the U area only serves to streamline the analysis and allows us to consider a homogeneous good for which the cross-market price constraint is unambiguous (R area buyers purchase the same good at the same price as U area buyers). As an alternative strategic mode, we could employ price setting (differentiated Bertrand). While, as noted below, this does not alter the basic strategic link between the U and R areas, it does introduce additional issues such as how to interpret the cross-market price constraint when products are differentiated.}
2.2 Benchmarks

Consider a monopolist operating only in area R. The monopolist would maximize $Q(1 - Q/b) - cQ$ by choosing output $b(1 - c)/2$ with price $(1 + c)/2$ and profit

$$\Pi^M = \frac{b(1 - c)^2}{4} - F^R$$

We assume that $F^R$ is sufficiently large for $\Pi^M < 0$. This assumption implies the need for subsidies if the government wants consumers in the R area to be served.

In addition we keep the result obtained in Anton et al. (2002) to be compared with ours. The first of key results is that the price in the rural and urban is $p = ((1 + b)(1 + c) + c)/(3 + 2b)$. Other key result is that the equilibrium bid when there is no outsider is $s = F^R - b(1 + b)^2(1 - c)^2/(3 + 2b)^2$. When an outsider is allowed to bid for the rural area, he bids as much $s_o = F^R - 2b(1 - c)^2/9$. Given this bid, the outsider wins the auction when the size of the rural area R, b, is sufficiently small.

3 Analysis

We proceed from the end of the game-tree back towards the beginning.

3.1 Step 1: pricing in the R area

It is straightforward that the $p^R_j \leq p^U$ is binding in equilibrium. In other words, the monopoly prices in the portions of the R area are higher than the equilibrium prices in the U market when one firm operates in both markets. Thus $p^R_j = p^U \forall j = 1, ..., M$.

3.2 Step 2: quantities supplied in the U area

Denote $q_{1i}$ the quantity supplied in portion $\alpha_i$ of the U area by the firm that only operates in the U that portion of the U area and by $q_{2i}$ the quantity supplied in portion $\alpha_i$ of the U area by the firm that operates in that portion

\footnote{The subsidy that the government has to pay to the winner is the fixed costs, $F^R$ plus the opportunity cost of serving the rural area.}
of the of the U area and in portion $\beta_j$ of the R area. Given the quantities supplied in the all the portions of the the U area, the prices are $p^U_i = (1 - (q_{1i} - q_{2i})/\alpha_i), i = 1, \ldots, N$. We know as well that these prices must be equal to the prices in the portions of the R area, $p^U_i = p^R_j, \forall i, j$. Then, market profits gross of fixed costs and subsidies are, for the firm that operates in portion $\alpha_i$ and in portion $\beta_j$ of the R area:

\[
\Pi_2 = (1 - \frac{(q_{1i} + q_{2i})}{\alpha_i} - c)\left(q_{2i} + \frac{\beta_j b(q_{1i} + q_{2i})}{\alpha_i}\right)
\]

and for the firm that operates only in the portion $\alpha_i$ of the U area:

\[
\Pi_1 = (1 - \frac{(q_{1i} + q_{2i})}{\alpha_i} - c)q_{1i}
\]

We can then derive the reaction functions:

\[
r^{1i}(q_{2i}) = \frac{\alpha_i(1 - c) - q_{2i}}{2} \quad \text{and} \quad r^{2i}(q_{1i}) = \frac{\alpha_i(1 - c)}{2} - \frac{(\alpha_i + 2\beta_j b)q_{1i}}{2(\alpha_i + \beta_j b)}
\]

which yield the equilibrium quantities:

\[
q_{1i}^* = \frac{\alpha_i(\alpha_i + \beta_j b)(1 - c)}{3\alpha_i + 2\beta_j b} \quad \text{and} \quad q_{2i}^* = \frac{\alpha_i^2(1 - c)}{3\alpha_i + 2\beta_j b}
\]

and the equilibrium mark-up is:

\[
p^U_i - c = 1 - \frac{(q_{1i}^* + q_{2i}^*)}{\alpha_i} = \frac{(\alpha_i + \beta_j b)(1 - c)}{3\alpha_i + 2\beta_j b}
\]

Given these results, we can conclude:

**Lemma 1** It can never be the case that the rural area is served by less firms than $N$, which the number of duopolies that serve the urban area.

We cannot face a situation where a number $K < N$ of firms, that operate in the urban area, operate in the rural area as well. This situation is not possible because we cannot maintain a uniform price across the different portions in which the urban area is divided. We would have got duopolies that have to compete more fiercely, duopolies without a firm serving a portion of the rural area, than others, duopolies with a firm serving a portion of the
rural area. This situation would bring to a situation where we do not have a uniform price across the urban area.

Moreover if we check the equilibrium price and quantities we can see that we can assume without lost of generality that the U area can be divided in symmetric portions, that is \( \alpha_i = \alpha_k = \alpha \), \( \forall i = 1, \ldots, N \), \( i \neq k \). From this assumption and the constraint that the price has to be the same across the different portions of the U and R area we can derive the following result:

**Proposition 1** If the U area is divided in symmetric portion, \( \alpha \). Then the R area covered by firms operating in the U area has to be divided in symmetric portion as well, \( \beta_j = \beta_s = \beta \), \( \forall j = 1, \ldots, N \), \( j \neq s \).

Given the results above, we can now summarize how the outcome in the urban and rural markets under the price constraints compares with the benchmarks. A direct comparison of the appropriate terms shows that:

**Proposition 2** Relative to the case where just a duopoly operates in the U area and one of these two firms serves the R area:

1. The equilibrium in the U area involves a higher price and a lower aggregate quantity when firms from duopolies do not cover the whole R area, \( \sum_1^N \beta < 1 \). Both, aggregate quantity and price, are equal under the condition that the firms from duopolies cover the whole R area, \( \alpha = \beta \).

2. The firms that operate in both markets supply a lower quantity when \( \alpha > \beta \). The opposite applies for the firms that just compete for the urban area. Both types of firms supply the same quantity when \( \alpha = \beta \).

3. If \( \sum_1^N \beta_j < 1 \) there are more firms serving in the R area than duopolies serving the U area, \( M > N \). If \( \alpha = \beta \), then all the R area is served by firms from the duopolies and each duopoly has a firm serving in the R area, \( M = N \).

To understand the result, it is good to remember that one of the firms of all duopolies is also serving in the R area as a monopolist. These firms face a tradeoff between trying to raise the price to get more profits from the rural area or losing profits from the urban area due to the competitive pressure of the other firm that also operates in the urban area. When \( \alpha = \beta \), the weights of the the U and R area are the same as when we consider just a
duopoly operating in the U area and one of these firms operating in the R area. As a consequence we obtain the same results about equilibrium price and aggregate quantity supplied to the U area than in Anton et al (2002).

If $\alpha < \beta$, there is more weight in the U area. The firms that operate in both areas compete more fiercely and that brings to a price drop. Comparing with the case where $\alpha = \beta$, the firm that just operates in the urban area serves more product to try to maintain the price as high as possible. It is good to point out that if $\alpha < \beta$, then we need extra firms to cover the whole R area, because the firms operating in the U area just cover a part of the R area.\(^{13}\)

**Lemma 2** The price decreases with $\alpha$. Moreover if $\alpha \to 0$ then the $p^U^* = p^R^* \to p^M$ where $p^M$ is the monopoly price.

The price decreases with $\alpha$ because as $\alpha$ rises, the urban area is more important for the firms of the duopolies that serve both areas, R and U. As we explain in the previous result. To become the U area more important makes competition in that area tougher with the final result of a lower price.

In the second part of the lemma it is shown that if the fixed costs do not matter in any portion of the U area, and the U area is divided in very small portions, then the price in the U area and R area tends to the monopoly price.\(^{14}\) This is because, in this case, for firms operating in the U and R area just matter the R area where they are monopolist.

**Lemma 3** The price grows with $\beta$. Furthermore, When $\beta \to 0 \forall j = 1, \ldots , N$ and $\beta_j \in [0, 1] \forall j = N + 1, \ldots, M \sum_1^M \beta_j = 1$ then $p^U^* = p^R^* \to p^C$ where $p^C$ is the price of cournot equilibrium if the rural area is not be served.

In this case, the price grows with $\beta$ because as $\beta$ the R area becomes more important respect to the U area as $\beta$ grows for the firms that operate in both markets. At this point it is good to remember that these firms are monopolist in the R area. As the R area is now more important, they are interesting in competing aggressively in the U area and in raising the price to enjoy more profit from the R area.

\(^{13}\)To sump up, if $\beta < \alpha$, where M is the total firms operating in the R area, and N is the total firms operating in the U area. Then, we need $M - N$ outside firms to cover the whole R area.

\(^{14}\)The monopoly price is expressed above in the benchmarks.
The other part of the lemma tells us that if the portions of the rural area that are served by firms operating in the urban area are so small that tend to 0, the price tends to the Cournot price equilibrium as if the rural area were not served. In this case, the portion of the rural area that has to be served by these firms is so small that they are just interesting in the U area and in competing with their rivals.

From the results we have got up to now we can derive the equilibrium profits. The equilibrium profit for the firms that operate only in the U market is:

\[ \Pi^*_1 = (p^*_i - c)q^*_1 - \delta_i F^U = \frac{\alpha_i(\alpha_i + \beta_j b)^2(1 - c)^2}{(3\alpha_i + 2\beta_j b)^2} - \delta_i F^U \]

and the equilibrium profit for the firm that operates in both markets is:

\[ \Pi^*_2 = (p^*_i - c)(q^*_2 + \frac{\beta_j b(q^*_1 + q^*_2)}{\alpha_i}) \Rightarrow \]

\[ \Rightarrow \Pi^*_2 = \frac{(\alpha_i + 2\beta_j b)^3(1 - c)^2}{(3\alpha_i + 2\beta_j b)^2} - \delta_i F^U - \gamma_j F^R \]

Finally, a direct comparison of these equilibrium profits with the equilibrium profits when there is just a duopoly serving the U area and a firm serving the U area and firm serving the R area yields the following result.

**Lemma 4** If \( \alpha > \beta \), the equilibrium profits of the duopolies are lower than when the U and R area were not divided. When \( \alpha = \beta \) the equilibrium profits do not change.

The result is just a direct consequence of Proposition 2.

### 3.3 Step 3: Equilibrium bids

**3.3.1 No Outsider is allowed to bid for portions of the R area**

If no outsider is allowed to enter in the R area, we are discussing the case where \( \alpha = \beta \). In this case the total payoffs of the bids \( s^A_i \) and \( s^B_i \) (and assuming equilibrium behavior in the continuation of play) in each portion are:
\[ v^x_i(s^x, s^y) = \begin{cases} 
\Pi^*_i & \text{if } s^x_i > s^y_i & x, y = A, B, \\
\Pi^*_2 + s^x_i & \text{if } s^x_i < s^y_i & i = 1, ..., N.
\end{cases} \]

where \( \Pi^*_i \) and \( \Pi^*_2 \) are calculated above. Note that the bids represent the required subsidies and an "aggressive" bid, one that increases the chance of winning the auction, is a low bid. Under the assumption that all the portions are not profitable the equilibrium bids are: \(^{15}\)

**Proposition 3** In each portion, in equilibrium each firm requires a subsidy equal to:

\[ s^*_i = \gamma_i F^R - \frac{\alpha b(1 + b)^2(1 - c)^2}{(3 + 2b)^2} / \]
\[ \sum_1^N s^*_i = F^R - \frac{b(1 + b)^2(1 - c)^2}{(3 + 2b)^2} \]

and wins the auction with probability \(1/2\). The total subsidy is equal to the subsidy asked in a market not fragmented.

This result makes sense because in aggregate terms the profits of losers and winners are the same in the two cases, when the market is fragmented and \( \alpha = \beta \) and when the market is not fragmented. \(^{16}\)

An important issue is the fact that is likely to have a portion where the subsidy is negative. In this case the auction of this portion is won by the firm with the highest bid. The equilibrium bid in this case is:

**Proposition 4** In equilibrium each firm pays for this portion:

\[ s^*_i = \frac{\alpha b(1 + b)^2(1 - c)^2}{(3 + 2b)^2} - \gamma_i F^R \]

and wins the auction with probability \(1/2\). In this case the total subsidy asked by all the portions, \( \sum_1^N s_i \), is lower than the subsidy asked when the market is not fragmented.

\(^{15}\)If the whole R area is not profitable, that does not always imply that the portion with the lowest fixed cost is not profitable.

\(^{16}\)The reader can check the results when the market is not fragmented in Anton et al. (2002).
The second part of the result is rather clear. As a portion of the R area is profitable, then we can save a part of the subsidy that would be given if the market were not fragmented. As the price is not affected by the number of portions in which the R and U areas are divided, then it may profitable to divide the U and R areas in as many portions as possible up to get a profitable portion in order to save in the aggregate subsidy asked by the firms that serve the R area.

In any case, a firm that operates only in the U area strictly prefers that its U area opponent also operates in the R area, because this makes the U area opponent a softer competitor in the U area. Thus, each firm is more "demanding" with the subsidy it requires.

The desire of the government to provide service to consumers in the rural area, in conjunction with the requirement that the rural price not exceed the urban price, can create a perverse incentive for each firm to lose the auction for the rural area in order to gain the more profitable position of serving only the urban market.

The effects described above are valid for much more general setting than the simple model presented here. In particular, they hold for standard Cournot models with nonlinear demand and costs. Further, the main effects are still present if there are more than two firms per portion of the U area, as long as these have market power (of course, as the market power of each firm in each portion decreases, the incentive to manipulate the price becomes weaker). Finally, the effect of the price restriction are not dependent on the specifics of quantity-setting as the strategic mode. Similar effects appear in a differentiated price-setting. The reason is the same as above, the reaction function of the firm operating in both areas is shifted in the direction (up in the case of price-setting) associated with being a "softer" competitor.

3.3.2 Outsiders are allowed to bid for portions of the R area

So far, we have focused in auctions between the firms that are active in the U area. We now introduce the possibility of entry in portions of the R area by firms that are not active in the U area, "outside" firms, for short. The critical difference is that outside firms cannot directly affect or manipulate the price in the U area.

\(^{17}\)A sufficient condition is to have downward sloping reaction function with a unique stable equilibrium.
An important issue in this case is how we design the auctions. We propose two types of auctions. In type I, in each portion of the U area, firms bid for a portion of the R area, once in all the duopolies one firm serves a portion the rest of portions are auctioned among the outsiders. In type II, the R area is just auctioned among the outsiders.18

Our inquiry includes the question of whether just inside incumbents or the combination of inside incumbents and outsiders or even just outsiders are more likely to request a smaller subsidy and win the auctions.

To allow the outside firms to participate guarantee us that $\beta$ is lower than $\alpha$.19 The first issue to be studied is the problem of having just one outside firm:

**Lemma 5** The subsidy that is required when there is one outsider and auction is type I or type II is much higher than when the market is not fragmented and insiders and outsiders compete for the R area.

The subsidies are higher because, in type I, a portion is just bid by the outsider. The same happens with the whole R area in the auction type II. In both cases the outsider bids the highest possible subsidy because it does not face any competition. The only interesting case where to have just one outsider is worthy is when there is just a duopoly in the U area and the firms of duopoly and the outsider compete for serving the R area.20 In this case the outsider puts pressure in the insider incumbents to gain the R area, having as a result the same or a lower subsidy comparing with the case where under the same scenario there is no outsiders.

We can conclude that if we have got just one outsider, it is not very convenient to use the type of auctions that are proposed in this paper, and we should focus on what is described in Anton et al. (2002).

The most relevant case is when there are $M \geq 2$ outsiders. First, we study the implications of using the type I auction. Under this type if we have $M$

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18 The reader can think why we do not consider an auction where the outsiders compete with the firms operating in the U area for the portions. The reason is that if we have less number of outsiders than the duopolies then the price across the different portions of the U area will be different. The case where we have at least the same number of outsiders as the duopolies is picked up with the auctions type I and II and auctions analyzed in Anton et al. (2002).

19 $\beta$ is size of the portion served by the firms that operate in the U market as well. $\alpha$ is size of the portion in the U area.

20 This case is treated deeply in Anton et al. (2002).
outsiders, then the number of portions in which the R area is divided is 
\( M - N - 1 \).\(^{21}\)

We have got \( M - 1 \) more portions when introducing the outsiders. From 
previous results we know that to “cut” the R area in more portions gives a 
lower price as a result and therefore, there is an improvement in consumers 
surplus. This improvement is higher as the number of outsiders grow, because 
the portions are smaller. In this context the equilibrium bids are:

**Proposition 5** If \( M \geq 2 \), in equilibrium:

1. each insider requires a subsidy:

\[
s^*_I = \gamma_j F^R - \frac{\beta b(\alpha + b\beta)^2(1 - c)^2}{(3\alpha + 2b\beta)^2} \quad j = 1, \ldots, N
\]

and wins the auction of a portion with probability \( 1/2 \).

2. each outsider requires a subsidy:

\[
s^*_O = \gamma_j F^R - \frac{\beta b(2\alpha + b\beta)(\alpha + b\beta)(1 - c)^2}{(3\alpha + 2b\beta)^2} \quad j = N + 1, \ldots, M - 1.
\]

and wins the auction of a portion with a probability \( 1 - 1/M \).

3. Compared with the case where there is no outsiders, if the number of 
outsiders is low \( \forall b > 0 \), the total subsidy is lower. but if the number 
of outsiders is sufficiently high, then if \( b \in [0, b'] \) the subsidy is lower 
and higher when \( b \in [b', \infty) \).

The insiders have to be compensated for the fixed costs of serving a 
portion of the R area plus the profit not won by not being a loser. The 
outsiders receive in equilibrium a subsidy that is lower than the fixed costs. 
This is because if an outsider loses the auction has 0 profit, but if an outsider 
wins the profit is the operating profits minus the fixed costs. The outsiders

\(^{21}\)The number of portions is the number of duopolies plus the number of outsiders minus one. This number guarantee that all the firms will bid as hard as possible. For further details, Milgrom (1987).
will bid up to be indifferent between to lose or to win, then the equilibrium subsidy is less than the fixed costs.

The third part of the proposition is due to two opposite forces. First, as $\beta < \alpha$, for the insiders, the rural area is less important than in the case where there are just insiders, and as the result the price is lower. That means that each insider, in equilibrium, asks for a higher subsidy. However, the outsiders that cover portions ask for a lower subsidy than the insiders, because they do not have the option of becoming a loser and winning a profit just competing in the U area. The first effect dominates the second when the number of outsiders is low and when the number is big and the R area is small. The second dominates when the number of outsider is high and the R area is big.

Therefore, when the R area is small, it is welfare enhancing to extend the number of portions of the R market to accommodate the outsiders for two reasons:

1. The consumer surplus raises because the market price is lower.

2. The total equilibrium subsidy asked by firms is lower compared to the case where there is no outsiders.

When the number of outsiders is high and the R area is big, the effect on welfare is ambiguous. On one hand the price is lower and therefore, the consumers surplus higher but on the other hand the subsidy is higher. The answer is not clear at this point and will be studied in following sections.

Next, we study the implications of using the type II auctions. It is good to remember, at this stage, that in this type of auction the R area is just auctioned among the outsiders.\textsuperscript{22}

As we restrict the auction just to the outsiders, the insiders do not serve the R area at all. This brings the consequence that, contrarily to the auction type I, here the number of outsiders do not influence the market price and it is the lowest possible.\textsuperscript{23}

In this case, the number of firms is not either important, if at least $M \geq 2$, for how big the portions should be. They can be of different sizes. As all the firms are equal they all bid, in equilibrium, the same amount for the

\textsuperscript{22}Again we suppose that there area $M \geq 2$ outsiders.

\textsuperscript{23}If there are insiders that serve both markets, the price is higher because the firms that serve in the R area are monopolist in that area and they try to keep the market price as high as possible to enjoy this monopolist position.
portions. We divide the R area in \( M - 1 \) portions, \( \beta_1, \ldots, \beta_{M-1} \geq 0 \) and \( \sum_{1}^{M-1} \beta_j = 1 \). In this scenario the equilibrium bids are:

**Proposition 6** If \( M \geq 2 \), in equilibrium, each outsider requires a subsidy:

1. 
   \[ s_j = F^R - \frac{\beta_j 2b(1-c)^2}{9}, \quad j = 1, \ldots, M - 1. \sum_{1}^{M-1} \beta_j = 1. \]
   
   and wins the auction for a portion with probability \( 1 - (1/M) \).

2. The total subsidy is:
   \[ s = F^R - \frac{2b(1-c)^2}{9} \]

   This subsidy is lower than the subsidy from auction type I when \( b \in [0, b^*] \), whereas it is higher when \( b \in [b^*, \infty] \).

The equilibrium bids are just the subsidy that makes each outsider indifferent between producing or not producing. The outsiders have to bid so hardly that the firms that win the auctions do not make any profit form serving in the R area. From the proposition, we also know that the subsidy required when using type II auction is lower when the R area is small than the subsidy required when using auction type I, and vice versa when the R area is big. Again, we find two opposite forces. On one hand, the opportunity cost for the insiders is higher, what induces a higher subsidy and on the other hand more market is served by outsider what induces a lower subsidy. The second force dominates when the R area is small because the price spread is very low. The opposite holds when the R area is big, because the price spread is much larger. Comparing the results obtained when using the auctions type II with those obtained when using auction type I we can conclude:

1. The price is lower with auction type II.

2. The subsidy is higher with auction type II when the R area is small and it is lower when the R area is big.
To conclude, if we compare the auction type II with the case where there is no outsiders, we can conclude:

1. The price is lower with auction type II. The reasons are the same as when we compared auction type I with the case where there is no outsiders.\textsuperscript{24}

2. If the R area is small then, the subsidy with auction type II is lower but from a size on, the subsidy is higher.\textsuperscript{25}

3.4 Step 4: Equilibrium portions $\alpha$ and $\beta$

The equilibrium portions depend on two decisions that have been chosen. Firstly, if the outsiders are allowed to enter to serve into the R area, and secondly if the outsiders are allowed to enter which type of auction we are going to use. In all these three cases, we are going to try to maximize Social Welfare. The Social Welfare is defined by the sum of consumers surplus minus the subsidies required to serve the R area.

3.4.1 Outsiders are not allowed to enter into the market

In this case, the issue about how to fragment the market is just important when we can get at least an isolate portion of the R area which is profitable. In such a case we should "cut" both areas to get as much profitable R area as possible. If we cannot get any profitable portion, it does not matter how we divide the market if the portions are symmetric.\textsuperscript{26} This is because the equilibrium profit functions do not depend on how we divide the U and R area.

3.4.2 Outsiders are allowed to enter into the market and auction type I

On the contrary of the previous case, if we allow outsiders to enter and we use the auction type I, the issue of how we fragment the market becomes very

\textsuperscript{24}The reader can refer for further explanations to the previous proposition.
\textsuperscript{25}The reasons why this occurs can be found in Proposition 4 in the paper by Anton et al. (2002).
\textsuperscript{26}In previous result we have shown that symmetric portions are necessary conditions to have the same price across the whole R and U area.
important. With this type of auction and given a number $M \geq 2$ outsiders, we find out a trade-off when dividing the U area between to have a low price of to have a low subsidy. If we divide the U area in many piece the subsidy is low but the price is maintained very close to the price with no outsiders. Of course, if we decide to place just a duopoly in the U area, the price is lower but the subsidy is higher. This tradeoff is solved in the following proposition:

**Proposition 7** Given $M \geq 2$ outsiders, then:

1. If the number of outsiders is high; $\forall b > 0$, the number of duopolies that is welfare maximizing is 1, $\alpha = 1$.

2. If the number of outsiders is low; $\forall b \in [0, \bar{b}]$ the number of duopolies that is welfare maximizing is 1, $\alpha = 1$. $\forall b \in [\bar{b}, \infty)$, the number of duopolies that is welfare maximizing is the highest possible.

If we have got many outsiders and just a duopoly we have a very low market price. If we think in dividing the U area in more portions there would be an increase in the market price because the size of the portions decrease much more in the U area than in the R area. On the contrary, the subsidy is lower because the operating profits are high due to the increase of the price but not enough to compensate the loss in consumers surplus.

The analysis becomes more complicated when the number of outsiders that are willing to enter is small. In this case, if we increase the number of insiders, the increase in the price is very important, this is because the size of the portions in both areas get smaller in a more proportioned way. At the same time, the subsidy is quiet lower because the operating profits raise. Finally to understand the result, we should remember that the price affects to both areas, then if the R area is small, any increase in the price never compensate the saving in the total subsidy. An increase in the number of firms in the U area just compensate when the R area is big enough to cover the decrease in the consumers surplus.

### 3.4.3 Outsiders area allowed to enter into the market and auction type II

Here, the issue of how to divide the area does not either influence the results in the following steps of the game but just in one case. It may be important, as in the case where the outsiders are not allowed to enter, if we can get
a portion that is profitable. In that case, that portion would be "cut" and auctioned among the outsiders. One important remark is that the portions do not have to be symmetric, because the outsiders do not influence directly the market price.

4 Welfare Analysis

Given the result so far, we have left to answer two last questions. The first question is: Should it be allowed the outsiders to enter into the market? and the second question: If we allow outsider to enter, Which type of auction should it be used?

We answer first the second question. If we analyze both types of auctions we find out that when the R area is small then the auction type II is welfare maximizing because it brings into the market the lowest possible price (highest consumers surplus) and the lowest subsidy. When the R area is bigger, then, on one hand, the price from auction type II is still the lowest but on the other hand, with auction type I, the subsidy required is lower. At first sight, it is not clear what is better.

If the number of outsiders is very high, the type II is welfare maximizing. This is because, the auction type I is served by almost just outsiders, the savings in subsidies can never compensate the loss in consumers surplus due to a high price. In this case, it is just important the R area. In any market where the demand is downward sloping if we increase the price, there is an increase in firms profits and a decrease in consumers surplus, being higher the decrease in consumers surplus.

If the number of outsiders is low, the auction type II is still better. The price from the auction type II is much lower because most of the R area is served by insiders. The high price also makes the subsidy lower when using auction type I, because firms enjoy higher operating profits. If we solve the tradeoff, we see that the gain in consumers surplus is higher than the save in the total subsidy. Therefore, if we allow outsiders to enter we should use auction type II to auction the R area.

Once we have answered the second question, we answer the second. To know if we allow the outsiders to enter into the market, we compare the results when using the auction type II with the results when not allowing outsiders to enter. If the R area is small is rather clear that auction type II is better for two reasons. First, the consumers surplus is higher and second,
the subsidy is lower. When the $R$ area is big, we face a tradeoff between to have a higher surplus, using auction type II, or to have a lower subsidy, using just insiders to cover the $R$ area. In this tradeoff, the final winner is the auction type II. To understand why this occurs, we should think just in terms of the $R$ area, because $U$ area is so small than it does not influence the result. As we explain before in this kind of market what is welfare maximizing is to choose the procedure that gets into the market the lowest market price.

5 Conclusions

In this paper, we have analyzed the strategic implications of the Universal Service Obligations when the profitable area is divided in independent areas (markets). We have studied how a regulator or policymaker should divide the profitable and unprofitable area to fit the requirements of the Universal Service, and at the same time maximizes the Social Welfare.

In this context, when the firms that operate in the profitable area are the only ones allow to serve the unprofitable area, we find that, given a division of the profitable market, the unprofitable area should be divided keeping the same proportions that in the profitable market due to the price constraints. Moreover, the price and total subsidy is exactly the same as when there is no division in any area. The result comes from the fact that in any of the subareas the competitive pressure is the same that when the areas are not divided.

When outside firms are allowed to bid, we have considered two types of auctions. The auction type I, where there is no auction where insiders and outsiders bid jointly. With this auction, the price is lower, because it is needed a number portion in the unprofitable area higher than in the profitable area. The insiders feel more competitive pressure because proportionally, they serve less unprofitable area where they are monopolist. The subsidy is also lower than when the outsiders are not allowed to enter when the number of outsiders is low. It is also lower when the number of outsiders is high and the unprofitable area is small but it is higher when the number of outsiders is high and the unprofitable area is big. When we try to maximize Social Welfare using this type of auctions, we face a tradeoff in the design of the subareas when the number of outsiders willing to enter is low. This tradeoff consists in dividing the profitable area in small pieces which implies that the unprofitable market is cut in small pieces as well, what gives a high price
and a low subsidy, or having a unique profitable market with a lower price and higher subsidy. This tradeoff is solved in a different way depending on how big is the unprofitable area. If it is small, the profitable area should not be divided but if it is big, it is better to divided it in as many subareas as possible. When the unprofitable area is small is more important to decrease prices because the subsidy is very low due to the size of the unprofitable area and viceversa when it is very big.

When the other type of auction is used, the inside firms are not allowed to bid. This brings to a situation where there is not any strategic link between the two areas, and therefore, the price is the lowest possible for this break of the strategic link. Another consequence from the break is that the price and subsidy do not depend on how any of the areas are divided.

Finally we have analyzed what should be done to maximize Social Welfare. If the unprofitable area is small, then, the problem becomes very easy because to allow outsiders to enter and to use auction type II give the lower price and subsidy. The answer becomes more complicated when the unprofitable area is big. In this case the auction type II is still the consumers surplus maximizing, but the other options have a lower subsidy. Taking into account this tradeoff we see that finally the option of allowing outsiders to enter and using the auction type II is always Social Welfare Maximizing.

Of course, more research on this topic is needed. The most interesting financial extension, at least for the case of telecommunications where there regulatory boards are applying this procedure, is to consider that the subsidy is covered by some tax on the firms operating in this industry, essentiality creating a cross-subsidy from the profitable area to the unprofitable area.

\footnote{We consider a high price, a price close to the case with no outsiders.}
References


