When Cheaper is Better: Fee Determination in the Market for Equity Mutual Funds

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February 12, 2004

Abstract

We provide evidence that equity mutual funds with worse before-fee performance charge higher fees. We then develop a model that explains this apparently anomalous finding. In the model, mutual fund managers of privately known, heterogeneous abilities compete to attract investors’ money. In equilibrium, better-performance funds never charge higher fees: if all investors react optimally to differences in expected performance, there is no fee dispersion; if some investors are relatively insensitive to differences in expected performance, however, worse-performance funds may set higher fees and serve only the less performance-sensitive investors.

*We would like to thank Sandro Brusco and seminar participants at Universidad Carlos III, Universitat Pompeu Fabra and the XXVIII Simposio de Análisis Económico for helpful comments. All remaining errors are our own. The authors gratefully acknowledge the financial support of Spain’s Ministries of Education and Culture (SEC 2001-1169) and of Science and Technology (BEC 2002-02194). Corresponding author: Pablo Ruiz Verdú, Universidad Carlos III de Madrid, Departament of Business Administration. Calle Madrid, 126. 28903 - Getafe, Madrid - Spain. E-mail: prverdu@emp.uc3m.es.
1 Introduction

In 2002, U.S. mutual fund assets were worth 6,392 billion dollars, and mutual fund holdings constituted an estimated 17.8 percent of the total financial wealth of U.S. households.\textsuperscript{1}

The increasing reliance of American investors on mutual funds has raised concerns among industry commentators and regulators alike about the level of fees charged by mutual fund management companies, prompting the General Accounting Office (GAO, 2000) and the Securities and Exchange Commission (SEC, 2000) to conduct reviews of mutual fund fee trends.\textsuperscript{2} Motivated by this debate, in this paper, we provide a theoretical model of the determination of fees in the mutual fund market.

Recent empirical findings underscore the need for a better theoretical understanding of the competitive forces behind mutual fund pricing. In particular, empirical studies of mutual fund performance have originated a number of intriguing questions about the working of the mutual fund market. Gruber (1996), for instance, documents performance persistence, mostly concentrated at the bottom of the performance distribution. This finding raises the question of why investors do not flee immediately to better-performing funds. The existence of differences in net (after-fee) performance also raises the question of why companies managing better-performing funds do not increase their fees up to the point at which investors are almost indifferent between their funds and worse-performing ones. As Gruber (1996) puts it, management ability does not seem to be priced in the mutual fund industry. In fact, Gruber’s (1996) results suggest that funds with better before-fee performance actually charge lower fees. More direct evidence of this relationship may be found in Carhart (1997) and Chevalier and Ellison (1999). In this paper, using data from Morningstar on a cross-section of U.S. equity open-end mutual funds, we confirm the existence of a negative significant association between before-fee performance and fees.

In the model we propose, funds of different qualities compete for investors’ money, and investors cannot observe fund quality before making their investment decision, an assumption that we believe characterizes well the mutual fund industry. The model shows\textsuperscript{1}

\textsuperscript{1}Investment Company Institute (2003).
\textsuperscript{2}Academics have echoed these concerns, and have studied different aspects of the distribution of fee levels in the mutual fund industry. See, for instance, Ferris and Chance (1987), Chance and Ferris (1991), Malhotra and McLeod (1997), Tufano and Sevick (1997), or Dellva and Olson (1998). Lesseig et al. (2002) and Golec (2003) provide more recent analyses.
that, if all investors react optimally to differences in expected returns, in equilibrium all participating funds should charge the same fees. Standard vertical differentiation outcomes, in which high-quality producers charge higher prices, do not apply to the mutual fund market, where the good being traded is monetary returns. Homogeneity in fees, however, need not be associated with homogeneity of returns. In fact, we show that, in equilibrium, good and bad funds coexist, which results in dispersion of net returns. Our model, thus, provides an equilibrium explanation for observed differences in after-fee performance across funds.

We extend the model to accommodate the presence of a pool of performance-insensitive or unsophisticated investors. The existence of these investors has been postulated by Gruber (1996) as an explanation for why money remains in funds that can be predicted to perform poorly and that, in fact, do perform poorly. Including these unsophisticated investors in the model changes the results greatly, as it allows for equilibria that not only display dispersion in after-fee performance, but in which fees and before-fee returns are negatively associated, a result consistent with our empirical results and those in Gruber (1996), Carhart (1997) and Chevalier and Ellison (1999).

The rest of the paper is organized as follows: section 2 presents the related theoretical literature; section 3 discusses the available evidence on the relationship between performance and fees; section 4 presents the model, which is extended in section 5 to include unsophisticated investors; and section 6 concludes.

2 Related Theoretical Literature

There exists a relatively large theoretical literature that attempts to characterize the optimal compensation contract in a delegated portfolio management problem.\(^3\) This literature adopts the view of an investor who wants to design a compensation contract to provide an agent - possibly of unknown ability - with the right incentives to manage the investor’s portfolio. In this article, however, we do not try to derive the optimal fund manager’s contract, but instead take the contract form that is standard in the industry as given: a

fee computed as a proportion of the value of the fund’s assets. Moreover, we assume that, in accordance with actual practice in the mutual fund industry, the level of the fee is set by the managing company. And, finally, we analyze the equilibrium of a market in which several mutual funds compete for the money of a large number of investors rather than looking at an investor and a fund manager in isolation. We believe that this approach is better suited to study fee patterns in the non institutional mutual fund industry, in which a large number of managing companies compete for the money of an even larger number of investors to whom they offer largely standardized contracts.

A related approach has been adopted by Hortacsu and Syverson (2003), who develop a search model of the market for S&P 500 index funds. In contrast to our paper, however, they analyze a sector in which financial performance differences across funds are relatively small and thus focus on non-portfolio fund differentiation and search frictions as potential sources of fee dispersion.

The role of informational asymmetries in related contexts has been explored by Metrick and Zeckhauser (1999) and Das and Sundaram (2002). Metrick and Zeckhauser (1999) attempt to explain why high- and low-quality producers may charge the same price in certain markets (including the mutual fund market). In the present paper, we also analyze fee setting in an asymmetric information context, although both our model’s assumptions and implications depart from theirs in crucial aspects, as we discuss in section 4.

Das and Sundaram (2002) compare the performance of two types of incentive schemes: fulcrum and incentive fees.4 To do so, they develop a model in which two fund managers compete for the money of a (representative) investor who cannot observe their quality. In their model, the compensation contract not only determines the fund manager’s incentives but may also signal the fund quality to the investor. By construction, however, only the high-ability manager is active in equilibrium, so their model is silent about the distribution of fees and returns.

Finally, in another closely related paper, Christoffersen and Musto (2002) show that

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4In both schemes, management fees are computed as a proportion of the fund’s net asset value. This proportion, however, depends on the performance of the fund relative to some benchmark. Fulcrum fees are symmetric in the sense that gains and losses of equal size with respect to the benchmark have exactly the same effect (though with opposite signs) on compensation. Incentive fees, in turn, are asymmetric since gains relative to the benchmark are compensated but losses are not penalized.
differences in the performance-elasticity of the demand curves faced by money-market funds may explain the differences in the fees charged by those funds. In our case, all funds compete to attract money flows from the whole pool of investors, performance-sensitive or not: the funds actions’ are what determine the performance sensitivity of their clientele.

3 Evidence on the Relationship between Performance and Fees

3.1 Mutual Fund Fee Structure

Investors pay two kinds of fees\(^5\): operating expenses and loads. Expenses mostly consist of management fees, but also include 12b-1 (distribution and marketing) fees, custody fees, administrative fees, operating, legal, and accounting costs, as well as other costs incurred by the fund each fiscal year. They are computed as a percentage of assets under management -termed “the expense ratio”- and are deducted on a daily basis from the fund’s net assets by the managing company.

Loads are generally used to pay distributors and they differ from operating expenses in that they are paid by the individual investor as a fraction of her investment at the time of purchasing fund shares (“sales charge on purchases”) or redeeming fund shares (“deferred sales charge”), whereas expenses are deducted directly from the fund’s assets under management. Consequently, since fund returns are typically computed from the fund’s net asset value, quoted returns are net-of-expenses, but before loads.

3.2 Previous findings

Starting with Hendricks, Patel, and Zeckhauser (1993), a number of empirical studies have confirmed the power of lagged fund returns to predict future performance in the short run. Gruber (1996), employing a sample of U.S. open-end non-specialized domestic mutual funds, finds that past lagged after-expense returns and risk-adjusted after-expense performance can be used to predict future performance. He further documents that this persistence in fund after-expense performance can be partly explained by the fact that superior management is not priced through higher expense ratios. Indeed, when ranking

\(^5\)For a more detailed description of mutual fund fee practices and regulation, we suggest that the reader visits the Online Publications section of the SEC internet site.
funds according to after-expense performance, Gruber finds that funds in the worst decile have the highest average expense ratio and that performance differences between the best and the worst funds exceed differences in fees. Together, these results suggest that worse funds—in terms of before-expense performance—may be charging higher fees.

Carhart (1997) shows, for non-sector non-balanced domestic equity funds, that a sizeable part of the predictive power of lagged returns with respect to future after-expense performance can be accounted for by portfolio composition. Consistently with Gruber (1996), however, he finds that the remaining after-expense persistence can be mostly explained by differences in expense ratios, which are especially high for funds in the worst performance decile. Furthermore, he estimates that funds with annual expenses of 100 basis points above the average correspond to funds with 154 basis points below average after-expense performance.

Finally, Chevalier and Ellison (1999), using a measure of performance similar to Carhart’s (1996), report that manager and fund characteristics—such as the portfolio turnover ratio and log of assets—contribute to explaining persistent differences in performance. Controlling for these variables, they provide estimates of the effect on after-expense performance of a 100 basis point reduction in expense ratios that range from 152 to 225 basis points.

Put together, the empirical evidence suggests that there is a significant degree of persistence in fund performance and that companies managing worse quality funds—i.e. funds with worse before-expense performance—do not charge lower fees to improve net returns to investors. In fact, it seems that funds with worse before-expense performance are more expensive, which further widens the gap in after-expense performance with respect to the best funds. Next, we explore whether the relationship between before-expense performance and expense ratios conforms to this pattern.

3.3 Data

We obtained Morningstar data on a cross-section of U.S. equity mutual funds from the MSN Money portal. Our initial dataset contained fund characteristics for January 2003 and time series statistics for the previous three years, such as the fund’s mean return or the fund’s beta. We restricted the dataset to include only non-institutional open-end
actively-managed mutual funds that had remained active in the full February 2000-January 2003 period.\textsuperscript{6} We also excluded specialty funds, so our final dataset contains only equity funds belonging to the following classes, established according to investment objectives: large growth, large blend, large value, mid-cap growth, mid-cap blend, mid-cap value, small growth, small blend, and small value.

Our dataset includes quantitative information on fees only for the last year of the sample, yet also reports whether a fund has changed its expense ratio in the preceding three years. Since we cannot follow the time-path of expense ratios and would need that information to obtain before-expense measures of performance, we further restricted the dataset to include only those funds that had not changed their expense ratios over the three-year period. Finally, we removed six outliers,\textsuperscript{7} so our final sample consists of 2,582 observations.

Table 1 displays descriptive statistics on fund expense ratios for our sample. As the table shows, there is wide dispersion in expense ratios across investment categories and across funds within the same class, with funds in the 90th percentile of any class charging fees more than twice as high as those charged by the funds in the 10th percentile, and with standard deviations within any class ranging from 44 to 56 basis points.

We use two proxies for fund performance provided by Morningstar: mean annualized monthly returns over the February 2000-January 2003 period ("mean return") and a measure of risk-adjusted performance, namely the fund’s Jensen’s alpha for the same period ("alpha"). Jensen’s alpha is the intercept from a least-squares regression of fund returns in excess of the Treasury Bill rate on excess returns on the market portfolio (S&P 500 index). It can therefore be interpreted as the part of the fund’s mean return not explained by the portfolio’s exposure to systemic risk.

Since reported returns are net of expenses, and we are interested in fund performance before expenses are deducted, we recovered performance gross of expenses from the data.\textsuperscript{8} Table 2 shows descriptive statistics for annualized performance measures—net and gross of expenses. Again, there is clear evidence of fund heterogeneity both across investment

\textsuperscript{6}Funds born after February 2000 or terminated before January 2003 were thus deleted.
\textsuperscript{7}The outliers were identified using Hadi’s multivariate outlier detection test. The variables included in Hadi’s test were before-expense alpha and expense ratio.
\textsuperscript{8}Appendix A.2 explains how we did this.
objectives and across funds with the same investment objective.

3.4 Expense ratios and before-expense performance

To analyze the relationship between performance and fees, we first estimate the equation:

\[ \text{PERF}_i = \gamma_0 + \sum_{k=2}^{K} \gamma_k \delta_{i,k} + \gamma_p \text{fee}_i + \varepsilon_i \]

where \( \text{fee}_i \) denotes the \( i \)-th fund’s expense ratio, \( \delta_{i,k} \) is a dummy variable that equals one if the \( i \)-th fund belongs to the \( k \)-th investment category, and \( \text{PERF}_i \) is the \( i \)-th fund’s before-expense performance as proxied by mean gross returns or gross alphas.

Table 3 displays estimated coefficients along with heteroscedasticity-robust p-values for both measures of performance. The results suggest that funds with expense ratios 100 basis points above class average are expected to generate mean before-expense returns (risk-adjusted returns) 87 (58) basis points lower than the class average, estimates clearly in line with those reported by both Carhart (1997) and Chevalier and Ellison (1999).

We check for the presence of nonlinearities in the relationship between fees and performance by analyzing the joint distribution of expense ratios and before-expense performance. Since average fees differ across investment categories, correlations between performance and expense ratios could be driven by differences in performance across investment categories during our particular sample period. To avoid this potentially confounding effect, we compute, for each of our performance variables, a measure of performance relative to the fund’s class average. This measure is defined as the fund’s excess performance with respect to the average for all funds with the same investment objective. We also compute funds’ relative expense ratios in the same fashion.

Table 4 illustrates the joint distribution of relative performance measures and relative expense ratios. For each performance measure, the deciles in table 4 are formed according to relative performance, with the first decile corresponding to funds with worst before-expense relative performance. Table 4 shows that, in line with the pattern noted in previous studies, there is a negative relationship between expense ratios and before-expense performance. Although the relationship is not perfectly monotonic, funds in the lowest performance deciles (for either measure of performance) charge expenses higher than those funds in the highest deciles. When we proxy performance by mean gross returns, we find that funds
in the worst decile are associated with expense ratios 4.85 basis points above their class average and that the best funds are the cheapest, with expenses 6.85 basis points lower than their class average. When performance is proxied by gross alphas, we find again that underperformers charge the highest fees, with a spread of 11 basis points in relative expense ratios between funds in bottom and top deciles. Table 4 also reports negative Spearman’s rank correlation coefficients for both performance measures that allow us to reject the null that expense ratios and performance are independent at the 1% significance level.9

To summarize, both regression results and Spearman’s rank test show that priciest funds not only fail to deliver returns that offset their higher cost, but even lag behind funds with lower fees in terms of before-expense performance. Results by Carhart (1997), and Chevalier and Ellison (1999) suggest that this finding is robust both to survivorship bias and to the inclusion of other explanatory variables in the regression equation.10

4 A Model of Fee Determination in the Market for Mutual Funds

Consider a simple setting in which there is a continuum of investors of mass one who have one dollar to invest, and \( N \) mutual fund managers.11 These managers can be of two types depending on their ability: good \((g)\) and bad \((b)\). G-managers earn gross expected return \( R_g \), and b-managers \( R_b \), where \( R_g > R_b \) and \( R_g > 1 \). The \textit{ex ante} distribution of types is given by the probability \( p \) that a manager is good. Once the types are realized, fund managers observe their quality but not the quality of their rivals and decide what fraction \( e \) of the fund’s final asset value to charge to investors. Investors do not observe quality, so that they decide where to invest on the basis of the prior distribution \( p \) and the fees charged by the different funds.

We assume that all market participants are risk-neutral, and that the only alternative investment is a risk-free asset paying zero interest rate.

9Spearman’s rank correlation coefficients are less sensitive than (Pearson) correlation coefficients to extreme values.
10In results not reported, we have also checked that the negative relationship between performance and fees extends to fees other than the expense ratio—such as front- and back-end loads—and to the sum of all fees.
11In our model, fund managers carry out all the tasks normally associated with managing companies, including the distribution of fund shares.
We also make the assumption that fund performance is independent of the fee charged by the fund, and thus leave aside all moral hazard problems. This assumption is made for tractability and to isolate the effects of asymmetric information about fund performance.

The costs of managing the fund are \( cw \), where \( w \) is the amount of money managed by the fund manager. We assume that costs are low enough to make it profitable for g-funds to operate if their type is known. The maximum fee a fund of type \( k \) can charge if its type is known is such that the net return for investors is equal to one: \( R_k(1 - e) = 1 \), that is \( e = \frac{R_k - 1}{R_k} \). Therefore, this assumption reads:

**Assumption 1** \( R_g - c - 1 > 0 \).

Note that Assumption 1 can be rewritten as \( \frac{R_g - 1}{R_g} > \frac{c}{R_g} \), where the right-hand side of the inequality is the break-even fee for g-funds.

We also assume that, given \( c \), b-funds may find it profitable to operate for some fee less than one hundred percent:

**Assumption 2** \( R_b - c > 0 \).

We denote by \( e_k \) the fee charged by a manager of type \( k \) as a proportion of the value of the fund at the end of the period and assume that there are no other fees. Therefore, the amount paid by an investor who invests \( w \) dollars in a fund of type \( k \) is \( we_k R_k \), and payoffs are \( w(e_k R_k - c) \) for the manager and \( w(1 - e_k)R_k \) for the investor.

The timing of decisions is as follows. First, managers simultaneously set fees. Then investors decide where to invest. We make the assumption that, if several funds have the same net expected returns, investors allocate their wealth among them with equal probability.

### 4.1 Benchmark Case: Complete Information

Before solving the model, it is instructive to investigate the relationship between fund quality and fees when quality is observed both by competing funds and by investors. It is straightforward to show that, in this case, b-funds will be driven out of the market whenever there are g-funds:
Proposition 1 With complete information, there do not exist equilibria in which both fund types are active and charge different fees.

Proof. First, note that for both types of funds to have a positive market share:

\[(1 - e_b)R_b = (1 - e_g)R_g\]  (1)

Suppose there is only one g-fund. For b-funds to operate \(e_b \geq \frac{c}{R_b} > \frac{c}{R_g}\), and, for any such \(e_b\) it is profitable for the g-fund to lower \(e_g\) slightly and attract the whole market. Therefore, at the only possible equilibrium \(e_g = 1 - (1 - c)\frac{R_b}{R_g}\), and b-funds do not operate.

If there are several g-funds, the same argument applies for any \(e_g > \frac{c}{R_g}\). The only possible equilibrium fee is \(e_g = \frac{c}{R_g} < \frac{c}{R_b}\), so b-funds remain inactive. \(\blacksquare\)

Therefore, the intuitive idea that good and bad funds can coexist as long as the latter charge lower fees does not hold in our model: whenever it is profitable for b-funds to operate, it is also profitable for g-funds to lower fees. As a result, b-funds are driven out of the market.

It should be noted that the literature on vertical differentiation (see Shaked and Sutton, 1982) has shown that equilibria in which low- and high-quality producers coexist, with the former charging lower prices, are possible. However, for this type of equilibria to exist, consumers must display differences in their willingness to pay for quality—because of differences in tastes or income.\(^{12}\) For sufficiently homogenous consumers, only high-quality producers survive.

In the mutual fund industry, the good provided by sellers is end-of-period dollars: a high-quality fund provides more end-of-period dollars per dollar invested. If end-of-period dollars were certain, all investors would value mutual fund quality equally, as nobody would pay more cents than anybody else for a dollar. In our model, the same argument applies, since we have assumed that investors are risk neutral. Therefore, the assumption of homogenous consumers regarding their willingness to pay for quality seems the natural one to make in this context.\(^{13}\)

\(^{12}\)For example, Metric and Zeckhauser (1999) present a model of vertical differentiation in which consumers differ in their taste for quality. Their model displays separating equilibria with high-quality producers charging higher prices.

\(^{13}\)The same reasoning can be extended to the case of risk-averse investors under standard assumptions. In particular, when the CAPM holds, all investors—indeed of their risk tolerance—will agree on the
4.2 Asymmetric Information

We first investigate whether there exist equilibria at which g- and b-managers set different fees (separating equilibria) and then turn to equilibria in which both manager types set the same fees (pooling equilibria). We use the Perfect Bayesian Equilibrium (PBE) as our equilibrium concept, and focus only on pure-strategy symmetric equilibria (i.e. equilibria in which all funds of the same type have the same equilibrium strategies). To limit equilibrium multiplicity, we require investors’ out-of-equilibrium beliefs to satisfy the property that they do not assign positive probability to managers setting fees that are certain to yield them a negative profit. That is, investors cannot assign a positive probability to a fund of type \( k \) choosing a fee less than \( \frac{c_{Rk}}{R_k} \). Therefore, throughout the paper, by *equilibrium* we will refer to a pure strategy Perfect Bayesian Equilibrium satisfying this restriction on investors’ beliefs.

First, note that in a separating equilibrium in which both types are ever active simultaneously, it has to be the case that net returns for investors are equal across types, since otherwise investors would not invest with the two types when both are available. Equality of net returns, in turn, implies that the expected market shares of g- and b-funds also have to be equal,\(^{14}\) since investors are indifferent between both types. But this implies that, if \( e_g > e_b \), it would be optimal for b-managers to imitate g-managers. On the other hand, if \( e_g < e_b \), no rational investor that observes both fees would invest with a b-manager. Therefore:

**Proposition 2** If all investors react optimally to differences in expected payoffs, there are no separating equilibria in which both fund-types operate simultaneously.

According to Proposition 2, we should observe no fee dispersion in equilibrium: for any realization of the number of b- and g-funds, all the funds with a positive market share must charge the same fees. The idea that b- and g-funds can be active simultaneously, with the latter charging higher fees, which was not supported in the complete information case, is not supported either when we allow for fund quality to be unobservable.

\(^{14}\)The expectation is taken over the possible realizations of the number of b- and g-funds.

maximum amount they would be willing to pay to invest with a given fund. Investors holding well diversified portfolios will want to invest with a fund as long as its net risk-adjusted returns are positive.
With asymmetric information, however, equilibria depart from the complete information, Bertrand-like outcome. The reason is that any g-fund faces a positive probability of competing only against b-funds. Therefore, there exists a strategy that guarantees positive expected profits to g-funds: setting a fee greater than the break-even fee for g-funds \((\frac{c}{R_g})\) but lower than the break-even fee for b-funds \((\frac{c}{R_b})\). Such a fee guarantees positive profits if the fund is ever able to attract any money and, as long as it is not too high, ensures that the fund would attract investors’ money at least when there are no competing g-funds. The fact that g-funds have a strategy that guarantees positive expected profits immediately implies that, in our model, an equilibrium with zero profits for g-funds is not possible. As the following proposition shows, this implies that there can be no separating equilibria. Proposition 2 ruled out equilibria in which b- and g-funds operate simultaneously while setting different fees. The following proposition strengthens this result. It shows that there are no separating equilibria and, thus, rules out equilibria like the ones obtained with complete information, in which g-funds drive b-funds out of the market.

**Proposition 3** If all investors react optimally to differences in expected payoffs, there are no separating equilibria.

**Proof.** First, we prove that g-funds must make positive profits in any equilibrium, for, if they made zero profits, there would always exist profitable deviations: a g-fund setting \(e \in (\frac{c}{R_g}, \frac{c}{R_b})\) would be identified as being of type g, and Assumption 1 guarantees that \(e < \frac{R_g - 1}{R_g}\) for e close enough to \(\frac{c}{R_g}\), so that investors would be willing to invest with the fund setting e, at least when all other funds are of type b. If we let \(w_b\) and \(w_g\) denote the wealth that b- and g-funds expect to obtain from investors if all funds play equilibrium strategies, this argument rules out equilibria with \(w_g = 0\) or \(e_g \leq \frac{c}{R_g}\).

Next, we show that, in any equilibrium \(e_g > \frac{c}{R_g}\). Suppose, on the contrary, that \(e_g \leq \frac{c}{R_g}\). Since g-funds must earn positive profits in equilibrium, \(e_g > \frac{c}{R_g}\). Now, consider a fee \(e = e_g - \epsilon\), with \(\epsilon > 0\), such that \(e > \frac{c}{R_g}\). A fund setting e would be identified as a g-fund and would capture the whole market with probability one. For \(\epsilon\) small enough, this is a profitable deviation.

Now, suppose that \(e_b = \frac{c}{R_b}\) or \(w_b = 0\), so b-funds earn zero profits. Since \(e_g > \frac{c}{R_b}\), it
would be profitable for them to imitate g-funds. Therefore, in any equilibrium, it must be the case that \( e_b > \frac{c}{R_b} \) and \( w_b > 0 \).

Finally, suppose that a separating equilibrium exists, and consider the following deviation for b-funds: \( e = e_b - \epsilon \) with \( \epsilon > 0 \). The fact that \( w_b > 0 \) implies that, for at least some realizations of the number of b- and g-funds, investors are willing to put their money with funds that charge \( e_b \) and are identified as b-funds. Therefore, they will be prefer to invest with a b-fund charging less than \( e_b \) at least for those realizations. This implies that, for any realization of the number of b- and g-funds for which b-funds’ market share is positive if they charge \( e_b \), the deviator would obtain the whole market. For \( \epsilon \) small enough, this is a profitable deviation. ■

The logic underlying Proposition 3 is the same that underlies the standard Bertrand outcome: at a separating equilibrium, b-funds cannot earn positive profits, for, otherwise, it would be optimal for a b-fund to slightly undercut \( e_b \). A b-fund that deviated in this way would be able to steal the whole market in all instances in which investors would have been willing to invest with funds–identified as b-funds–charging \( e_b \). However, an equilibrium in which b-funds earn zero profits is not possible, implying that there are no separating equilibria.

If both fund types set the same fee \( (e_p) \) in equilibrium, however, investors will believe that any fund setting \( e_p \) is of type g with probability \( p > 0 \). A deviating fund thus runs the risk of being interpreted by investors as being worse than those setting \( e_p \) and, therefore, runs the risk of losing all market share even if it sets a fee below \( e_p \). The next proposition shows that, if investors’ beliefs are sufficiently pessimistic when they observe a deviating fund, there can exist equilibria in which both funds set the same fee and obtain positive profits.

**Proposition 4** For some parameter values, there exist (pooling) equilibria in which both types set \( e_p \geq \frac{c}{R_g} \).

**Proof.** See appendix. ■

Figure 1 shows that pooling equilibria can exist for a broad range of reasonable parameter values. In the figure, the region below each curve represents the set of values of \( p \) and \( c \).
for which pooling equilibria can exist for given values of $R_g$ and $R_b$. The figure also shows how increasing $p$ expands the range of values of the other parameters for which pooling equilibria exist. Reducing $c$ or $N$ has a similar effect.\footnote{In the proof of Proposition 4, the conditions for existence are derived explicitly. From those conditions, it is straightforward to derive this comparative statics result.}

Proposition 4 shows that the presence of asymmetric information can limit competition among funds allowing for equilibria in which both fund types coexist and earn positive profits, an outcome that could not arise under complete information.

The existence of an equilibrium at which funds of different qualities set the same fee has already been proposed by Metrick and Zeckhauser (1999). Their model, however, differs from ours in a number of fundamental dimensions, and, most importantly, the mechanism that allows for a pooling outcome in their paper is different from ours. Metrick and Zeckhauser (1999) study a vertically-differentiated duopoly characterized by sequential price setting (with good funds setting fees–front-end loads–before bad funds) and by investor heterogeneity along two dimensions: on the one hand, different investors value the “good” provided by mutual funds differently; and, on the other hand, some investors can observe quality, while others cannot. In this context, an equilibrium in which both funds set the same price can arise when the qualities are similar enough. The reason is that competition for the investors who can observe quality is strong in this case. So strong that the good fund may find it optimal to set a fee so low that it is not profitable for the bad fund to set a fee that would convince at least some of the informed investors to buy from it. In such case, the bad fund may prefer to pool and give up all informed investors. It is important to note that, in their model, good funds attract more money than bad funds in a pooling equilibrium, since the latter do not get any money from informed investors. It is also worth noting that, in their model, there are also separating equilibria in which both funds are active and good funds charge higher fees. In our model, these equilibria are not possible.

5 Unsophisticated Investors

To explain why a sizeable proportion of investors holds underperforming funds, Gruber (1996) has proposed that there is a non-negligible proportion of unsophisticated investors who do not react optimally to differences in fund returns. In the U.S., investors can choose
from thousands of funds. To pick the optimal fund or set of funds, an investor would, in principle, have to evaluate the expected performance of each fund using all relevant information available. This may be beyond the capabilities of a significant fraction of investors and, even for the financially knowledgeable ones, involves costly search effort. Investors could alternatively buy the services of financial advisers, but doing so is also costly, and the quality of the advice received may be hard to evaluate. Therefore, many investors, especially those investing small sums, may not necessarily put their money in the ex ante optimal funds. These investors may be content to invest in funds (selected because of advertising, advice from acquaintances or brokers, or other reasons) as long as they do not have reasons to believe that they are obvious underperformers.

In this section, we extend the model to incorporate the possibility that not all investors are able to gather or interpret correctly all available information or to move their money fast enough when differences in expected returns are identified. We capture this possibility in the simplest form possible by assuming that a fraction $\gamma < 1$ of investors are unsophisticated. To reflect the idea that unsophisticated investors do not perform a full search among all available funds, we assume that each unsophisticated investor is paired with a mutual fund at random, although our results would generalize to the case in which unsophisticated investors observe only a relatively small number of fees. Once paired with a fund, however, unsophisticated investors do not invest blindly: they invest only if the fee charged by the fund is not “too high”. Instead of proposing a particular model of how boundedly rational investors decide what “too high” means, we denote by $e_U$ the maximum fee that unsophisticated investors are willing to pay and treat this maximum fee as a parameter. Each fund thus captures $\frac{\gamma}{N}$ dollars from unsophisticated investors as long as it sets a fee not greater than $e_U$.

The presence of unsophisticated investors can significantly alter the results in previous sections. Intuitively, it may allow for equilibria in which low- and high-quality funds operate simultaneously and set different fees, if each type of fund serves a different investor segment.

Note that the result in Proposition 2 would still hold in this case: in equilibrium, b-managers and g-managers cannot both serve the sophisticated market segment and charge different fees. If $e_g > e_b$, b-managers would mimic g-managers’ pricing strategy. If $e_g < e_b$,
sophisticated investors would not invest in b-funds. Therefore, if the presence of unsophisticated investors allows for the existence of separating equilibria in which both fund types are active simultaneously, sophisticated investors must prefer one type of fund over the other. This implies that expected net returns for b-funds and g-funds are different in equilibrium. One possibility is that b-funds offer a higher net return, which obviously requires $e_b < e_g$. However, it is straightforward to show that there cannot exist separating equilibria in which g-funds serve only unsophisticated investors and b-funds serve sophisticated investors. As argued in the proof of Proposition 3, such equilibria would necessarily imply that b-funds earn zero profits, in which case any b-manager could profitably deviate by setting a higher fee and serving unsophisticated investors only.

We investigate next whether there can exist equilibria in which both fund types are active simultaneously and charge different fees, with b-funds serving only unsophisticated investors as long as there are competing g-funds. The following conditions must hold at this type of equilibrium:

$$w^U_g (R_g e_g - c) \geq w^U_b (R_g e_b - c) \quad (\text{NIgU})$$
$$w^U_b (R_b e_b - c) \geq w^U_g (R_b e_g - c) \quad (\text{NIbU})$$
$$w^U_b (R_b e_b - c) \geq 0 \quad (\text{PbU})$$

where $w^U_k$ is the wealth that a fund setting $e_k$ expects to obtain conditional on all other funds playing the equilibrium strategies. If b-funds serve only unsophisticated investors whenever there are g-funds, $w^U_g > w^U_b$.

The first two conditions are no-imitation constraints for g- and b-funds, respectively, and the last condition is a participation constraint for b-funds. A participation constraint for g-funds is not necessary, because it is implied by (NIbU) and (PbU). Note that, since $w^U_g > w^U_b$, condition (NIbU) requires $e_g < e_b$: in this type of equilibrium, g-funds must set lower fees.

Fees also have to be low enough to convince both sophisticated and unsophisticated to participate:

$$e_g \leq \frac{R_g - 1}{R_g} \quad (2)$$
$$e_b \leq e_U \quad (3)$$
Finally, it cannot be profitable for b- or g-funds to deviate and set an out-of-equilibrium fee. To evaluate these deviations, we assume that investors’ beliefs are as described in the previous section: any deviation from equilibrium is interpreted as coming from a b-fund unless it yields negative profits for such a fund. The next proposition shows that there are parameter values such that all the above conditions hold simultaneously and there are no profitable out-of-equilibrium deviations:

**Proposition 5**  For $e_U > \frac{R_b - 1}{R_b}$, there exist separating equilibria with unsophisticated investors at which:

1. b-funds serve unsophisticated investors only and charge $e_b^{**} = e_U$

2. g-funds charge $e_g^{**} > \frac{e}{R_g}$ and serve both sophisticated and unsophisticated investors.

3. $e_b^{**} > e_g^{**}$

**Proof.** See appendix.

Figures (2)-(4) show that separating equilibria of this sort can exist for reasonable parameter values. The figures graph the minimum and maximum values of $c$ (plotted along the y-axis) for which these equilibria can exist for each possible value of $\gamma$ (plotted along the x-axis) for the case in which $e_U = \frac{\bar{R} - 1}{\bar{R}}$, where $\bar{R}$ is the unconditional expected gross return. This particular value of $e_U$ would obtain if unsophisticated investors had correct prior beliefs about the distribution of types and did not interpret fees as signals of fund quality.

The model in this section departs from the benchmark complete information model in two dimensions, and it is instructive to see how each of these dimensions contributes to the existence of separating equilibria like the ones described in Proposition 5. First, the existence of unsophisticated investors allows b-funds to survive while setting fees that differ from those of g-funds. As we saw in the previous section, this would not be possible if all investors held correct beliefs in equilibrium and could move their money freely. Second, the presence of asymmetric information limits the competitive pressure on g-funds. If sophisticated investors could observe fund quality, competition among g-funds would drive
e_g down to \( \frac{e}{R_b} \), but such a situation could not be an equilibrium with unsophisticated investors, because g-funds can set a higher fee, sell to unsophisticated investors and make a positive profit. It should thus be emphasized that the existence of unsophisticated investors alone cannot generate separating equilibria with both fund types active.

Relaxing the assumption that there are only two fund types would not change the results. If there were several fund types, some of them would sell to unsophisticated investors only and the rest to both sophisticated and unsophisticated ones. Those selling to unsophisticated investors would still charge the maximum possible fee, so that there would be pooling in the low part of the distribution of types. There would also be pooling in the upper part, as, if there was separation, some funds would be charging higher fees than others while still attracting sophisticated investors, which cannot be sustainable in equilibrium.

The proof of Proposition 5 shows that a separating equilibrium exists only if:

\[
e_U > \frac{R_b - 1}{R_b},
\]

where \( \frac{R_b - 1}{R_b} \) is the fee that guarantees the reservation return when investing with a b-fund. Therefore, at this type of equilibrium, some unsophisticated investors (those paired with b-funds) would do better by investing in the reservation asset.\(^{16}\) A number of realistic—and not exclusive—assumptions about the behavior of unsophisticated investors would yield this result. First, unsophisticated investors may fail to fully understand the equilibrium relationship between fees and gross returns, that is, they may—at least partly—fail to interpret fees as signals of fund quality. Second, they may not account properly for the effect of fees on net returns. In a recent regulatory proposal by the SEC (SEC, 2002) that would require mutual funds to provide a clearer disclosure of the dollar value of the fees paid by investors, one can read: “Despite existing disclosure requirements and educational efforts, the degree to which investors understand mutual fund fees and expenses remains a significant source of concern.” The proposal provides information from a previous report (SEC/OCC, 1996) that found “that fewer than one in five fund investors could give any estimate of expenses for their largest mutual fund and fewer than one in six fund investors understood that higher expenses can lead to lower returns.” Similar concerns have been voiced by

\[^{16}\text{This implies that, at these equilibria, we are effectively not requiring unsophisticated investors to hold the right beliefs and to maximize their expected utility given their beliefs and the observed fee.}\]
the General Accounting Office in a report (GAO, 2000) whose principal conclusion was that additional disclosure would help to increase investor awareness and understanding of mutual fund fees. Third, unsophisticated investors could be over-optimistic with respect to fund returns. Finally, unsophisticated investors could be small investors who face returns from the alternative investment lower than those of larger investors, either because of economies of scale in investing or higher interest rates for large investments. According to this last interpretation, unsophisticated investors would be fully rational, yet behave differently because of worse alternative investment opportunities.

6 Discussion

In this paper, we have shown that, in the mutual fund industry, better-quality sellers should not be expected to charge higher prices. Moreover, investors’ limited ability to evaluate fund quality may lead to equilibria in which worse-performing funds charge higher fees. We thus obtain a form of reverse price differentiation consistent with the evidence presented here and in previous empirical work on mutual fund performance.

Our analysis suggests several directions for further research. First, in our model we have considered a single period, so investors cannot base their decisions on past fund performance. An intertemporal extension of the model would allow one to investigate the relationship between fees and past performance, and their relative role as signals of fund quality. Second, we have assumed that costs are linear and equal for all funds regardless of their quality. It would be interesting to relax this assumption and allow for more general cost structures and for different correlations between costs and returns. Moreover, while we have assumed fund quality to be exogenous, mutual fund management companies may, to some extent, set the quality of the funds they offer through their choice of managers or their expenditure in market analysis. We have also assumed that unsophisticated investors are equally likely to buy from good and bad funds. However, funds could differentiate themselves not only through fees but also through their marketing decisions: in a separating equilibrium, lower-quality funds may not only charge higher fees, but also invest more in their distribution networks or advertising to make sure that they attract a larger proportion of unsophisticated investors.
Results in this paper indicate that the complexity associated with the evaluation of funds’ net expected returns may, on the one hand, mute the competitive pressure in the mutual fund industry, leading to higher fees and lower average returns, and, on the other hand, lead to a segmented market in which a number of investors may end up paying higher fees as well as obtaining lower returns. Whether this state of things could be improved by regulation and the optimal form of this regulation are questions that merit further scrutiny. A possibility, pursued by the SEC, would be to require that funds improve their disclosure of past performance and that they present in their prospectuses gross and net returns separately, thus highlighting the effect of expenses. Results also suggest that some funds may be overcharging investors and, thus, open the question as to whether some form of fee cap could be beneficial in this context. Recent judiciary initiatives in this direction, like the settlement reached by a mutual fund company and the New York attorney general by which the former agreed to cut management fees by an average of 20% highlight the need for more research in this area.\textsuperscript{17}

\textsuperscript{17}Brewster (2003).
References


A Appendix.

A.1 Proofs

Proof of Proposition 4.

Let $e_p$ be the pooling fee and $\overline{R} = pR_g + (1 - p)R_b$ denote the unconditional expectation of gross returns. We will assume that investors’ beliefs are such that, if a fund sets $e \in \left[ \frac{c}{R_g}, \frac{c}{R_b} \right)$, it will be believed to be a g-fund, while for any $e \geq \frac{c}{R_b}$ other than the equilibrium fee, it will be believed to be a b-fund.

For investors to be willing to buy from a fund of unknown type:

$$e_p \leq 1 - \frac{1}{\overline{R}}$$

(PCip)

For b-funds to be willing to participate:

$$e_p \geq \frac{c}{R_b}$$

(PCbp)

Let $m_g$ and $m_b$ be the minimum fees that would make it profitable for a g- or a b-fund, respectively, to deviate.

$$(m_g R_g - c) = \frac{1}{N} (e_p R_g - c)$$

$$(m_b R_b - c) = \frac{1}{N} (e_p R_b - c)$$

It follows that:

$$m_g = \frac{1}{N} e_p + (1 - \frac{1}{N}) \frac{c}{R_g} < \frac{1}{N} e_p + (1 - \frac{1}{N}) \frac{c}{R_b} = m_b,$$

so that if it is not profitable for a g-fund to deviate, neither it is for a b-fund. Now, let $\varepsilon$ be the maximum fee that would convince investors to shift to a fund believed to be bad, that is:

$$(1 - \varepsilon)R_b = (1 - e_p)\overline{R},$$

or

$$\varepsilon = 1 - (1 - e_p)\frac{\overline{R}}{R_b} \quad (4)$$

Therefore, if a fund deviates and sets $d \equiv \max\{\varepsilon, \frac{c}{R_b}\}$, it will capture the whole market,\(^{18}\) so, for a g-fund not to be willing to deviate, it has to be the case that:

$$m_g \geq d,$$

\(^{18}\)Or, rather, $d - \varepsilon$, where $\varepsilon > 0$ can be arbitrarily small.
which also implies that b-funds do not want deviate because \( m_b > m_g \).

**Case 1: \( d = \xi \).** Let us first look at the case in which \( d = \xi \), that is:

\[
1 - (1 - e_p) \frac{R}{R_b} \geq \frac{c}{R_b}, \quad \text{or} \quad e_p \geq \tilde{e} \equiv \frac{R - (R_b - c)}{R}
\]  

(5)

In this case the no-deviation condition for g-funds reads:

\[
m_g = \frac{1}{N} e_p + \left(1 - \frac{1}{N}\right) \frac{c}{R_g} \geq 1 - (1 - e_p) \frac{R}{R_b} = e, \quad \text{or} \quad e_p \leq \frac{N R_g (R - R_b) + (N - 1) R_b c}{R_g (N R - R_b)}
\]  

(6)

For an equilibrium of this sort to exist, thus, conditions (PCip), (PCbp), (5) and (6) have to hold simultaneously.

First note that, given Assumption 2, (5) implies (PCbp). Inspection of the conditions also shows that for (PCip) and (5) to hold simultaneously it is necessary that

\[
R_b - c > 1
\]  

(7)

If this condition holds, then it only rests to check that (5) and (6) can hold simultaneously. This requires:

\[
\frac{R - (R_b - c)}{R} < \frac{N R_g (R - R_b) + (N - 1) R_b c}{R_g (N R - R_b)}
\]  

(8)

After some algebra, this condition can be shown to be equivalent to:

\[
R_b < p \left( R_b + R_g \left( \frac{R_b - N c}{(N - 1)c} \right) \right)
\]  

(9)

Therefore, if \( R_b > N c \), a pooling equilibrium will exist for high enough values of \( p \).

**Case 2: \( d = \frac{c}{R_b} \).** In equilibrium, \( d = \frac{c}{R_b} \) if and only if:

\[
e_p \leq \frac{R - (R_b - c)}{R}
\]  

(10)

For g-funds not to deviate, we need \( m_g \geq d = \frac{c}{R_b} \), i.e.:

\[
e_p \geq \frac{c}{R_g} + N c \frac{R_g - R_b}{R_g R_b}
\]  

(11)
Thus, for an equilibrium of this sort to exist, conditions (PCip), (PCbp), (10), and (11) must hold. We need to consider two cases:

1. $R_b - c > 1$. In this case, condition (PCip) is implied by (10) and conditions (PCbp) and (10) are always compatible:

$$\frac{\bar{R} - (R_b - c)}{\bar{R}} > \frac{c}{R_b} \iff \bar{R}R_b - R_b(R_b - c) > c\bar{R} \iff$$

$$\bar{R}(R_b - c) - R_b(R_b - c) > 0 \iff \bar{R} - R_b > 0,$$

which is always true.

It rests to check that conditions (10) and (11) are compatible as well. This will happen if and only if:

$$\frac{c}{R_g} + Nc \frac{R_g - R_b}{R_g R_b} < \frac{\bar{R} - (R_b - c)}{\bar{R}} \quad (12)$$

Rearranging this expression leads to inequality (9), so the same conditions as above guarantee existence of this type of equilibrium.

2. $R_b - c < 1$. Now, condition (10) is implied by (PCip). The latter condition will be consistent with (11) only if:

$$1 - \frac{1}{\bar{R}} > \frac{c}{R_g} \left(1 + \frac{NR_g}{R_b} - N\right) \quad (13)$$

For fixed $R_b$ and $R_g$, the supremum of the left-hand side is $1 - \frac{1}{\bar{R}}$ (when $p \to 1$). The infimum of the right-hand side is $\frac{R_b - 1}{R_g} \left(1 + \frac{NR_b}{R_b} - N\right)$ if $R_b > 1$ (when $c \to 1 - R_b$), and 0 if $R_b < 1$ (when $c \to 0$). In the latter case, the above condition will hold. If $R_b > 1$, we must have:

$$\frac{R_g - 1}{R_g} > \frac{R_b - 1}{R_g} \left(1 + \frac{NR_g}{R_b} - N\right) \quad (14)$$

Rearranging,

$$(R_g - 1)R_b > (R_b - 1)(R_b + NR_g - NR_b) \iff$$

$$R_b < \frac{N}{N - 1} \quad (16)$$

Therefore, if (16) holds and $\frac{R_g - 1}{R_g} > \frac{c}{R_b}$, then there are pooling equilibria with $R_b - c < 1$. ■
Proof of Proposition 5.

First, notice that \( e_b \leq \frac{R_b-1}{R_b} \) cannot be an equilibrium fee, as slightly undercutting such \( e_b \) would guarantee the deviating b-fund all the sophisticated market in case there are no g-funds and would only marginally reduce its profits in all other cases. This implies that, in equilibrium \( e_b > \frac{R_b-1}{R_b} \), so a necessary condition for the existence of a separating equilibrium is \( e_U > \frac{R_b-1}{R_b} \). In what follows, we assume this to be the case.

Next, notice that, if a separating equilibrium exists, \( e^{**}_b = e_U \). Any \( e_b \in (\frac{R_b-1}{R_b}, e_U) \) cannot be an equilibrium, as such a fee will not convince sophisticated investors to invest with a b-fund even if all funds turn out to be of type b, and \( e_U \) yields greater profits from the unsophisticated investors. Since \( e_b \leq \frac{R_b-1}{R_b} \) cannot be an equilibrium fee either, the only possible equilibrium fee for b-funds is \( e^{**}_b = e_U > \frac{c}{R_b} \).

Given \( e_b = e_U > \frac{R_b-1}{R_b} \), \( w^U_b = \frac{\gamma}{N} \), so the participation constraint for b-funds and the no-imitation constraints read:

\[
\frac{\gamma}{N}(R_be_U - c) \geq 0 \quad \text{(PbU)}
\]

\[
\frac{\gamma}{N}(R_be_U - c) \geq w^U_b(R_be_g - c) \quad \text{(NIbU)}
\]

\[
w^U_g(R_ge_g - c) \geq \frac{\gamma}{N}(R_ge_U - c), \quad \text{(NIgU)}
\]

where \( w^U_g \in (\frac{\gamma}{N}, \frac{\gamma}{N} + (1 - \gamma)) \).

The no-imitation constraints can be rewritten:

\[
e_g \geq \frac{\gamma}{Nw^U_g}e_U + (1 - \frac{\gamma}{Nw^U_g}) \frac{c}{R_g} = \alpha e_U + (1 - \alpha) \frac{c}{R_g} \quad \text{(NIgU')}\]

\[
e_g \leq \frac{\gamma}{Nw^U_g}e_U + (1 - \frac{\gamma}{Nw^U_g}) \frac{c}{R_b} = \alpha e_U + (1 - \alpha) \frac{c}{R_b} \quad \text{(NIbU')}\]

where \( \alpha = \frac{\gamma}{Nw^U_g} \). Since \( w^U_g > \frac{\gamma}{N}, \alpha < 1 \). Therefore, the incentive constraint (NIbU') implies that \( e_g < e_U \), which proves part 3.

Let us assume that sophisticated investors' beliefs are such that if a fund sets \( e \in [\frac{c}{R_g}, \frac{c}{R_b}] \), it will be believed to be a g-fund, while for any \( e \geq \frac{c}{R_b} \) (other than g-funds' equilibrium fee if \( e_g \geq \frac{c}{R_b} \)), it will be believed to be a b-fund. This implies that \( e_g \in (\frac{c}{R_g}, \frac{c}{R_b}] \) cannot be an equilibrium fee. Since \( e_g \) needs to be strictly greater than \( \frac{c}{R_b} \), the only possible equilibrium fees satisfy \( e_g > \frac{c}{R_b} \).
Let $m_b$ be the minimum fee that would make it profitable for a b-fund to deviate if it captures the whole sophisticated market:

\[
\left(\frac{\gamma}{N} + (1 - \gamma)\right) (R_b m_b - c) = \frac{\gamma}{N} (R_b e_U - c), \quad \text{i.e.,}
\]

\[
m_b = \frac{\gamma}{N - \gamma(N - 1)} e_U + \left(1 - \frac{\gamma}{N - \gamma(N - 1)}\right) \frac{c}{R_b} = \lambda e_U + (1 - \lambda) \frac{c}{R_b},
\]

(17)

where $\lambda \equiv \frac{\gamma}{N - \gamma(N - 1)} < 1$.

Similarly, let $m_g$ be the minimum fee that would make it profitable for a g-fund to deviate if it captures the whole sophisticated market:

\[
\left(\frac{\gamma}{N} + (1 - \gamma)\right) (R_g m_g - c) = w_g^U (R_g e_g - c)
\]

(18)

Rearranging:

\[
m_g = \frac{N w_g^U}{\gamma + (1 - \gamma) N e_g} + \left(1 - \frac{N w_g^U}{\gamma + (1 - \gamma) N}\right) \frac{c}{R_g} = \phi e_g + (1 - \phi) \frac{c}{R_g},
\]

(19)

where $\phi \equiv \frac{N w_g^U}{\gamma + (1 - \gamma) N} < 1$.

Let $M_b$ ($M_g$) be the the minimum fee that would make it profitable for a b-fund (g-fund) to deviate and capture the whole sophisticated market only when there are no g-funds:

\[
\frac{\gamma}{N} (R_b e_U - c) = (R_b M_b - c) \left(\frac{\gamma}{N} + (1 - p)^{N-1}(1 - \gamma)\right)
\]

(20)

\[
w_g^U (R_g e_g - c) = (R_g M_g - c) \left(\frac{\gamma}{N} + (1 - p)^{N-1}(1 - \gamma)\right)
\]

(21)

Notice that these inequalities imply $m_g < M_g$ and $m_b < M_b$.

If $e_g > \frac{c}{R_b}$, the maximum fee that a deviating fund can charge while guaranteeing the whole sophisticated-investor market with probability one is $d \equiv \max\{\hat{e}, \frac{c}{R_b}\}$, where

\[
\hat{e} \equiv \frac{e_g R_g - (R_g - R_b)}{R_b}
\]

(22)

is defined by $(1 - e_g) R_g = (1 - \hat{e}) R_b$.

Similarly, the maximum fee that a deviating fund can charge while guaranteeing the whole sophisticated-investor market in case all other funds are of type b is

\[
D \equiv \max\{\frac{c}{R_b}, \frac{R_b - 1}{R_b}\}
\]

(23)
Therefore, the no-deviation conditions for b- and g-funds are, respectively:

\[ m_b \geq d \quad \text{(NDbU)} \]
\[ m_g \geq d \quad \text{(NDgU)} \]
\[ M_g \geq D \quad \text{(NDgU')} \]
\[ M_b \geq D \quad \text{(NDgU')} \]

Finally, \( e_g \) has to be such that sophisticated investors are willing to invest with g-funds:

\[ e_g \leq \frac{R_g - 1}{R_g} \quad \text{(PIgU)} \]

which will immediately hold as long as \( e_U \leq \frac{R_g - 1}{R_g} \), since \( e_g < e_U \).

An equilibrium will exist if all the inequality conditions (PbU, NIbU’, NIgU’, NDbU, NDgU, NDbU’, NDgU’, and PIgU) are satisfied simultaneously.

Given the relatively large number of parameters \((R_g, R_b, p, N, c, \gamma)\) and inequalities, we do not fully characterize the set of equilibria. Instead, we next show existence numerically. Figures 2–4 show parameter regions for which this type of equilibrium exists for the case \( e_U = \frac{R}{R-1} \), with \( R = pR_g + (1 - p)R_b \).

**A.2 Recovery of Before-expense Performance**

In order to add back expenses, we make the approximation that expenses are subtracted from the fund’s final net asset value at the end of each month. Since expense ratios have remained constant in the sample period for the funds in our sample, mean before-expense monthly returns can be approximately recovered from mean after-expense monthly returns as follows:

\[ 1 + Mean \ \text{Gross Return} = \frac{1 + Mean \ \text{Return}}{1 - \text{Expense ratio}} \quad (24) \]

To recover before-expense risk-adjusted returns, we would ideally use data on fund gross returns and then perform a regression of each fund’s gross excess returns on the market portfolio excess returns in the usual fashion:

\[ R_{\text{gross},t} - R_{ft} = \alpha_{\text{gross}} + \beta_{\text{gross}}(R_{mt} - R_{ft}) + \epsilon_{gt} \]
where \( R_{\text{gross},t} \) is the fund’s one-period gross return, \( R_{ft} \) the gross risk-less interest rate, and \( R_{m} \) the gross return on the market portfolio. The estimated intercept (Jensen’s alpha) is the manager’s abnormal return if the CAPM holds. It is computed as:

\[
\hat{\alpha}_{\text{gross}} = \bar{R}_{\text{gross}} - \bar{R}_{f} - \hat{\beta}_{\text{gross}}(\bar{R}_{m} - \bar{R}_{f})
\]

where \( \bar{R}_{\text{gross}}, \bar{R}_{f}, \) and \( \bar{R}_{m} \) are sample averages, and \( \hat{\beta}_{\text{gross}} = \frac{\text{cov}(R_{\text{gross},t}, R_{m} - R_{ft})}{\text{var}(R_{m} - R_{ft})} \), which is approximately \( \frac{\text{cov}(R_{\text{gross},t}, R_{m})}{\text{var}(R_{m})} \) if the risk-free return is approximately constant.

Since we do not have data on before-expense fund returns, we must obtain before-expense alphas from after-expense alphas. To do this, note that:

\[
R_{\text{net},t} - R_{ft} = \alpha_{\text{net}} + \beta_{\text{net}}(R_{m} - R_{ft}) + \epsilon_{nt}
\]

And hence,

\[
\hat{\alpha}_{\text{net}} = \bar{R}_{\text{net}} - \bar{R}_{f} - \hat{\beta}_{\text{net}}(\bar{R}_{m} - \bar{R}_{f})
\]

Assuming expenses are deducted from the fund’s assets at the end of each month, \( R_{\text{net},t} = R_{\text{gross},t}(1 - e) \), and hence \( \hat{\beta}_{\text{net}} = \frac{\text{cov}(R_{\text{net},t}, R_{m})}{\text{var}(R_{m})} = \hat{\beta}_{\text{gross}}(1 - e) \), so:

\[
\hat{\alpha}_{\text{net}} = \bar{R}_{\text{gross}}(1 - e) - \bar{R}_{f} - \hat{\beta}_{\text{gross}}(1 - e)(\bar{R}_{m} - \bar{R}_{f})
\]

\[
= \hat{\alpha}_{\text{gross}} - [(\bar{R}_{\text{gross}} - \hat{\beta}_{\text{gross}}(\bar{R}_{m} - \bar{R}_{f})]e
\]

Substituting \( \bar{R}_{\text{gross}} = \bar{R}_{f} + \hat{\alpha}_{\text{gross}} + \hat{\beta}_{\text{gross}}(\bar{R}_{m} - \bar{R}_{f}) \) in the last equation:

\[
\hat{\alpha}_{\text{net}} = \hat{\alpha}_{\text{gross}} - (\bar{R}_{f} + \hat{\alpha}_{\text{gross}})e
\]

So,

\[
\hat{\alpha}_{\text{gross}} = \frac{\hat{\alpha}_{\text{net}} + \bar{R}_{f} \cdot e}{1 - e}
\]
## B Tables

### Table 1. Descriptive Statistics: Expense Ratios

<table>
<thead>
<tr>
<th>Fund Class</th>
<th>Number</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Coeff. of Variation</th>
<th>75th to 25th %ile Ratio</th>
<th>90th to 10th %ile Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Growth</td>
<td>522</td>
<td>1.60</td>
<td>0.50</td>
<td>0.3125</td>
<td>1.6667</td>
<td>2.2727</td>
</tr>
<tr>
<td>Large Blend</td>
<td>546</td>
<td>1.48</td>
<td>0.56</td>
<td>0.3783</td>
<td>1.7944</td>
<td>2.7000</td>
</tr>
<tr>
<td>Large Value</td>
<td>436</td>
<td>1.52</td>
<td>0.51</td>
<td>0.3355</td>
<td>1.7727</td>
<td>2.5833</td>
</tr>
<tr>
<td>Medium Growth</td>
<td>338</td>
<td>1.69</td>
<td>0.50</td>
<td>0.2958</td>
<td>1.6328</td>
<td>2.1574</td>
</tr>
<tr>
<td>Medium Blend</td>
<td>99</td>
<td>1.56</td>
<td>0.53</td>
<td>0.3397</td>
<td>1.6000</td>
<td>2.2100</td>
</tr>
<tr>
<td>Medium Value</td>
<td>121</td>
<td>1.55</td>
<td>0.44</td>
<td>0.2838</td>
<td>1.5410</td>
<td>1.9630</td>
</tr>
<tr>
<td>Small Growth</td>
<td>267</td>
<td>1.76</td>
<td>0.48</td>
<td>0.2727</td>
<td>1.5556</td>
<td>2.0171</td>
</tr>
<tr>
<td>Small Blend</td>
<td>124</td>
<td>1.68</td>
<td>0.56</td>
<td>0.3333</td>
<td>1.6627</td>
<td>2.4554</td>
</tr>
<tr>
<td>Small Value</td>
<td>129</td>
<td>1.66</td>
<td>0.47</td>
<td>0.2831</td>
<td>1.6640</td>
<td>2.0909</td>
</tr>
<tr>
<td>ALL</td>
<td>2582</td>
<td>1.59</td>
<td>0.52</td>
<td>0.3200</td>
<td>1.6667</td>
<td>2.3684</td>
</tr>
</tbody>
</table>

### Table 2. Descriptive Statistics: Fund Performance (annualized, in %)

<table>
<thead>
<tr>
<th>Fund Class</th>
<th>Mean Return</th>
<th>Alpha</th>
<th>Mean Gross Return</th>
<th>Gross Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
<td>Mean</td>
<td>S.D.</td>
</tr>
<tr>
<td>Large Growth</td>
<td>-21.10</td>
<td>6.36</td>
<td>-5.69</td>
<td>4.86</td>
</tr>
<tr>
<td>Large Blend</td>
<td>-13.47</td>
<td>6.09</td>
<td>-0.39</td>
<td>4.68</td>
</tr>
<tr>
<td>Large Value</td>
<td>-5.22</td>
<td>4.45</td>
<td>5.97</td>
<td>5.08</td>
</tr>
<tr>
<td>Med. Growth</td>
<td>-18.15</td>
<td>9.67</td>
<td>0.59</td>
<td>8.01</td>
</tr>
<tr>
<td>Med. Blend</td>
<td>-4.54</td>
<td>6.88</td>
<td>9.48</td>
<td>7.71</td>
</tr>
<tr>
<td>Med. Value</td>
<td>2.89</td>
<td>6.11</td>
<td>14.74</td>
<td>7.06</td>
</tr>
<tr>
<td>Small Blend</td>
<td>1.32</td>
<td>6.46</td>
<td>14.85</td>
<td>6.76</td>
</tr>
<tr>
<td>Small Value</td>
<td>6.68</td>
<td>4.81</td>
<td>17.50</td>
<td>5.29</td>
</tr>
<tr>
<td>ALL</td>
<td>-11.55</td>
<td>10.91</td>
<td>3.01</td>
<td>9.30</td>
</tr>
</tbody>
</table>
### Table 3. Expense Ratios and Performance (OLS)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Mean Gross Return Coefficient</th>
<th>p-value</th>
<th>Gross Alpha Coefficient</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-18.4321</td>
<td>0.0000</td>
<td>-3.1644</td>
<td>0.0000</td>
</tr>
<tr>
<td>Large Blend</td>
<td>7.5432</td>
<td>0.0000</td>
<td>5.1847</td>
<td>0.0000</td>
</tr>
<tr>
<td>Large Value</td>
<td>15.9949</td>
<td>0.0000</td>
<td>11.7267</td>
<td>0.0000</td>
</tr>
<tr>
<td>Medium Growth</td>
<td>3.1602</td>
<td>0.0000</td>
<td>6.5438</td>
<td>0.0000</td>
</tr>
<tr>
<td>Medium Blend</td>
<td>16.7804</td>
<td>0.0000</td>
<td>15.3702</td>
<td>0.0000</td>
</tr>
<tr>
<td>Medium Value</td>
<td>24.3022</td>
<td>0.0000</td>
<td>20.6806</td>
<td>0.0000</td>
</tr>
<tr>
<td>Small Growth</td>
<td>6.6379</td>
<td>0.0000</td>
<td>11.1095</td>
<td>0.0000</td>
</tr>
<tr>
<td>Small Blend</td>
<td>22.9723</td>
<td>0.0000</td>
<td>21.0413</td>
<td>0.0000</td>
</tr>
<tr>
<td>Small Value</td>
<td>28.3679</td>
<td>0.0000</td>
<td>23.6913</td>
<td>0.0000</td>
</tr>
<tr>
<td>Expense Ratio</td>
<td>-0.8707</td>
<td>0.0010</td>
<td>-0.5853</td>
<td>0.0150</td>
</tr>
</tbody>
</table>

Observations 2582
R-squared 0.5837
F test 573.6 0.0000

### Table 4. Relative Performance and Relative Expense Ratios. Funds Ranked According to Relative Performance

<table>
<thead>
<tr>
<th>Decile</th>
<th>Mean Gross Return</th>
<th>Gross Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Performance</td>
<td>Exp. Ratio</td>
</tr>
<tr>
<td>Worst 1</td>
<td>-11.8842</td>
<td>0.0485</td>
</tr>
<tr>
<td>2</td>
<td>-6.1700</td>
<td>0.1091</td>
</tr>
<tr>
<td>3</td>
<td>-4.0384</td>
<td>0.0381</td>
</tr>
<tr>
<td>4</td>
<td>-2.4614</td>
<td>0.0191</td>
</tr>
<tr>
<td>5</td>
<td>-1.0767</td>
<td>-0.0318</td>
</tr>
<tr>
<td>6</td>
<td>0.2168</td>
<td>0.0190</td>
</tr>
<tr>
<td>7</td>
<td>1.7326</td>
<td>-0.0577</td>
</tr>
<tr>
<td>8</td>
<td>3.6633</td>
<td>-0.0425</td>
</tr>
<tr>
<td>9</td>
<td>6.3236</td>
<td>-0.0330</td>
</tr>
<tr>
<td>Best 10</td>
<td>13.6140</td>
<td>-0.0684</td>
</tr>
</tbody>
</table>

SRC\(^b\) \(-0.0879^*\) \(-0.0587^*\)

\(^a\) All data annualized in %. Table entries report average values by decile relative to each fund’s class average

\(^b\) Spearman Rank Correlation Coefficient

\(^*\) Significant at 1\% level
C Figures

Figure 1: Existence of Pooling Equilibria. \( R_b = 1.1; R_g = 1.3 \).

Figure 2: Conditions for Existence of Separating Equilibria with Unsophisticated Investors. \( R_b = 1.1; R_g = 1.3; p = 0.5; N=2 \). The x-axis displays values of \( \gamma \), while \( c \) is displayed along the y-axis. The curves represent the minimum and maximum values of \( c \) such that a separating equilibrium exists for each \( \gamma \).
Figure 3: Conditions for Existence of Separating Equilibria with Unsophisticated Investors. $R_b = 1.1; R_g = 1.3; p = 0.5; N=5$. The x-axis displays values of $\gamma$, while $c$ is displayed along the y-axis. The curves represent the minimum and maximum values of $c$ such that a separating equilibrium exists for each $\gamma$.

Figure 4: Conditions for Existence of Separating Equilibria with Unsophisticated Investors. $R_b = 1.1; R_g = 1.3; p = 0.5; N=10$. The x-axis displays values of $\gamma$, while $c$ is displayed along the y-axis. The curves represent the minimum and maximum values of $c$ such that a separating equilibrium exists for each $\gamma$. 